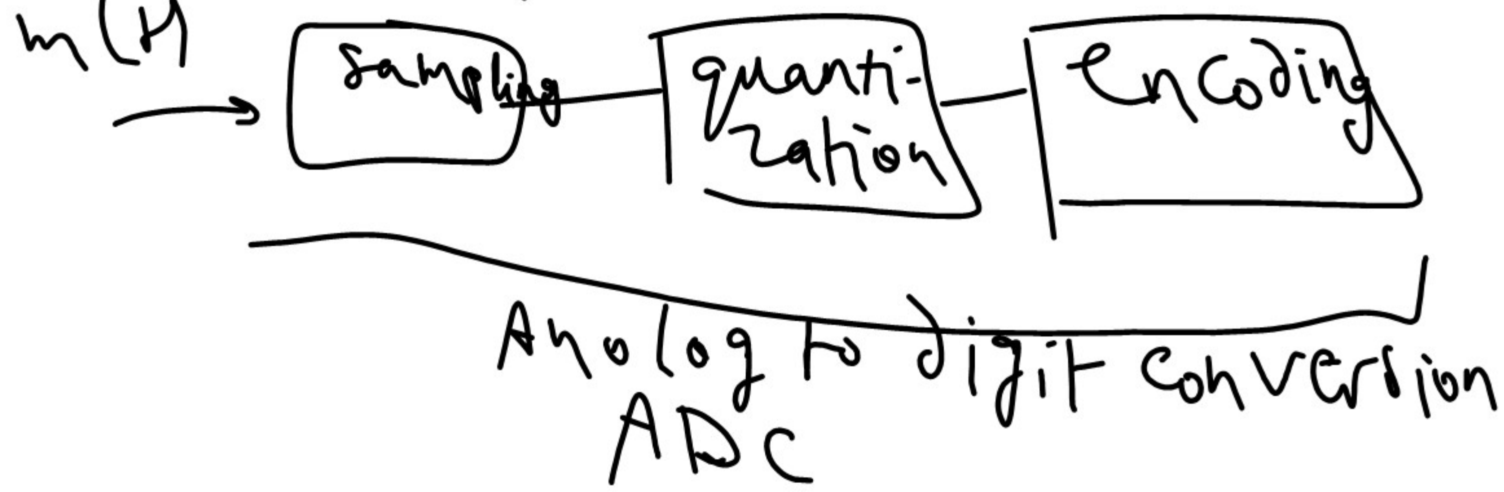
Quantization

* Quantization is the process of converting discrete-time continuous amplitude signal discrete-time discrete amplitude signal
* quantization is used either in communiction or in processing signals via digital computer
* quantizer are followed by an encoder
* The signal that results at the encoder output is kwon as pulse code modulation PCM
* The following three operations



Quantizers can be classifieds into uniform and nonuniform

Uniform quantization

* in uniform quantizers the decision between two Levels wich is called the step size Δ is the same between all levels

$$\Delta = \frac{V_p + V_p}{2^R} = \frac{2V_p}{2^R}$$

V_p is the peak amplitude of the signal to be quantized

R is the number of bits

Non Uniform quantizers

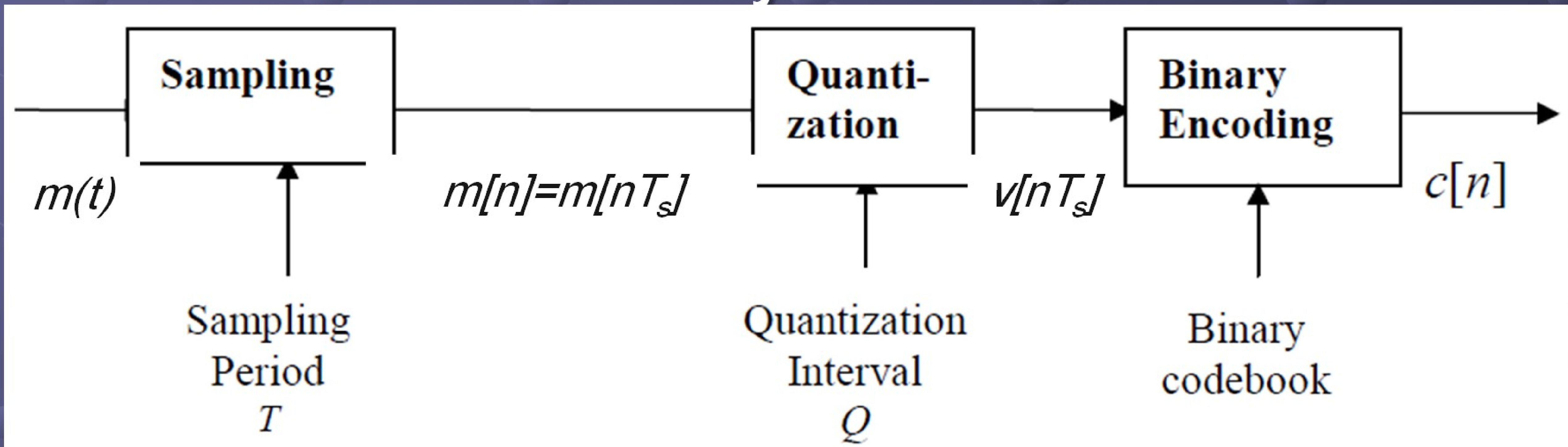
* In non uniform quantizers, the step size is made variable between the differen levels
* Nonuniform quantization can be used to represent voice signals because, it was found by reseach that human voice contains samples with very magnitude and samples with very large magnitude
* In order to represent the samples with Low magnitude we need small $\Delta \to$ large R
* on the other hand If we made Δ Large to represent the samples with large magnitude the the samples with low magnitude will not be presented properly

Analog to digital conversion

- The first step to move from analog to digital communications is to digitize the analog signal
- This can be accomplished by using an analog to digital converter A/D
- The analog to digital converter is composed from three different processes

Analog to digital A/D converter

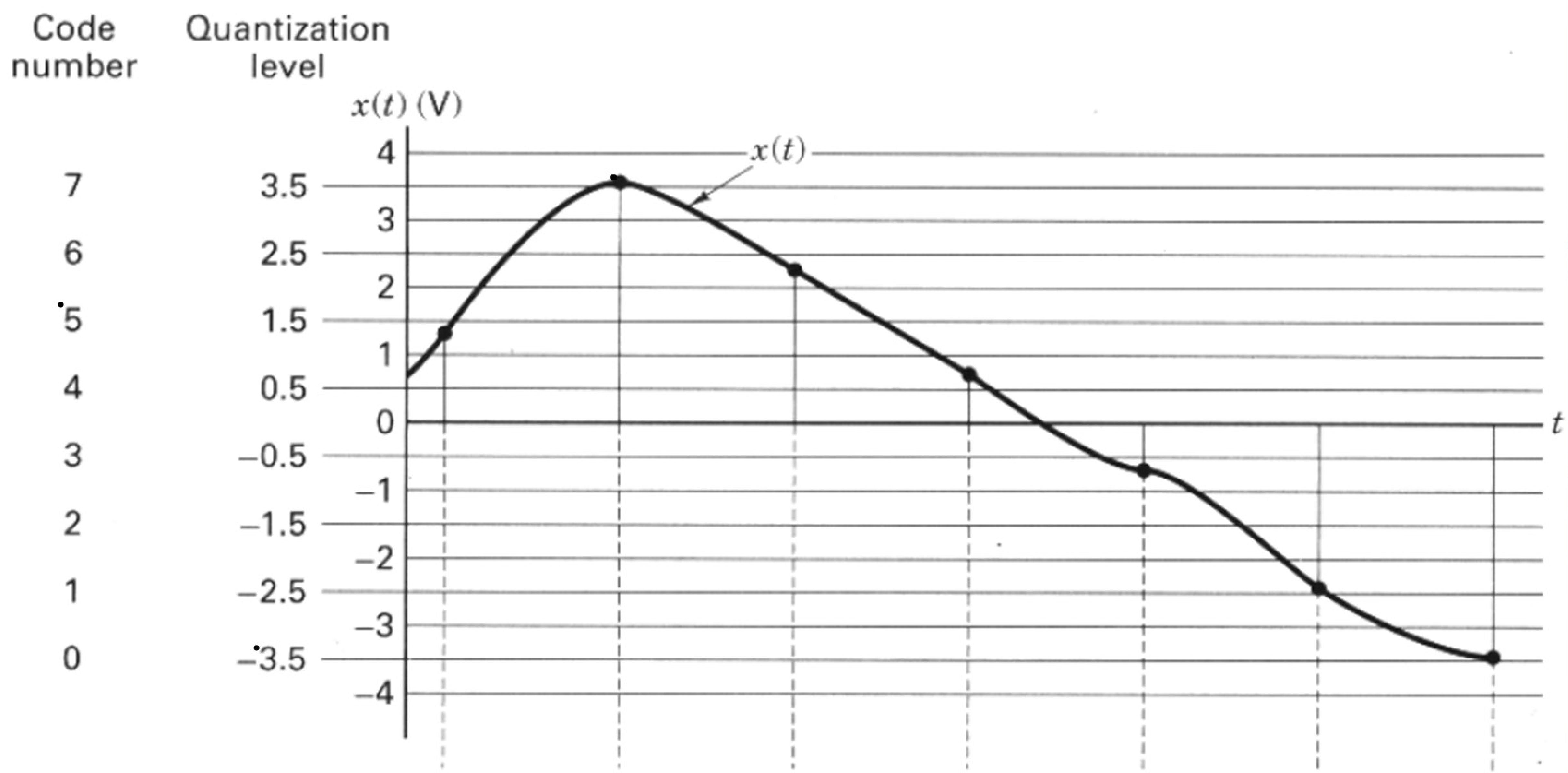
1. Sampling, which is explained in details
2. Quantization process
3. Binary encoding, convert each quantized value into a binary code word



Quantization

- Quantization is the process of converting continuous amplitude samples $m(nT_s)$ of a continuous message signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible amplitudes
- This process is illustrated graphically in the next slide

Quantization



Natural sample value	1.3	3.6	2.3	0.7	–0.7	–2.4	–3.4
Quantized sample value	1.5	3.5	2.5	0.5	–0.5	–2.5	–3.5
Code number	5	7	6	4	3	1	0
PCM sequence	101	111	110	100	011	001	000

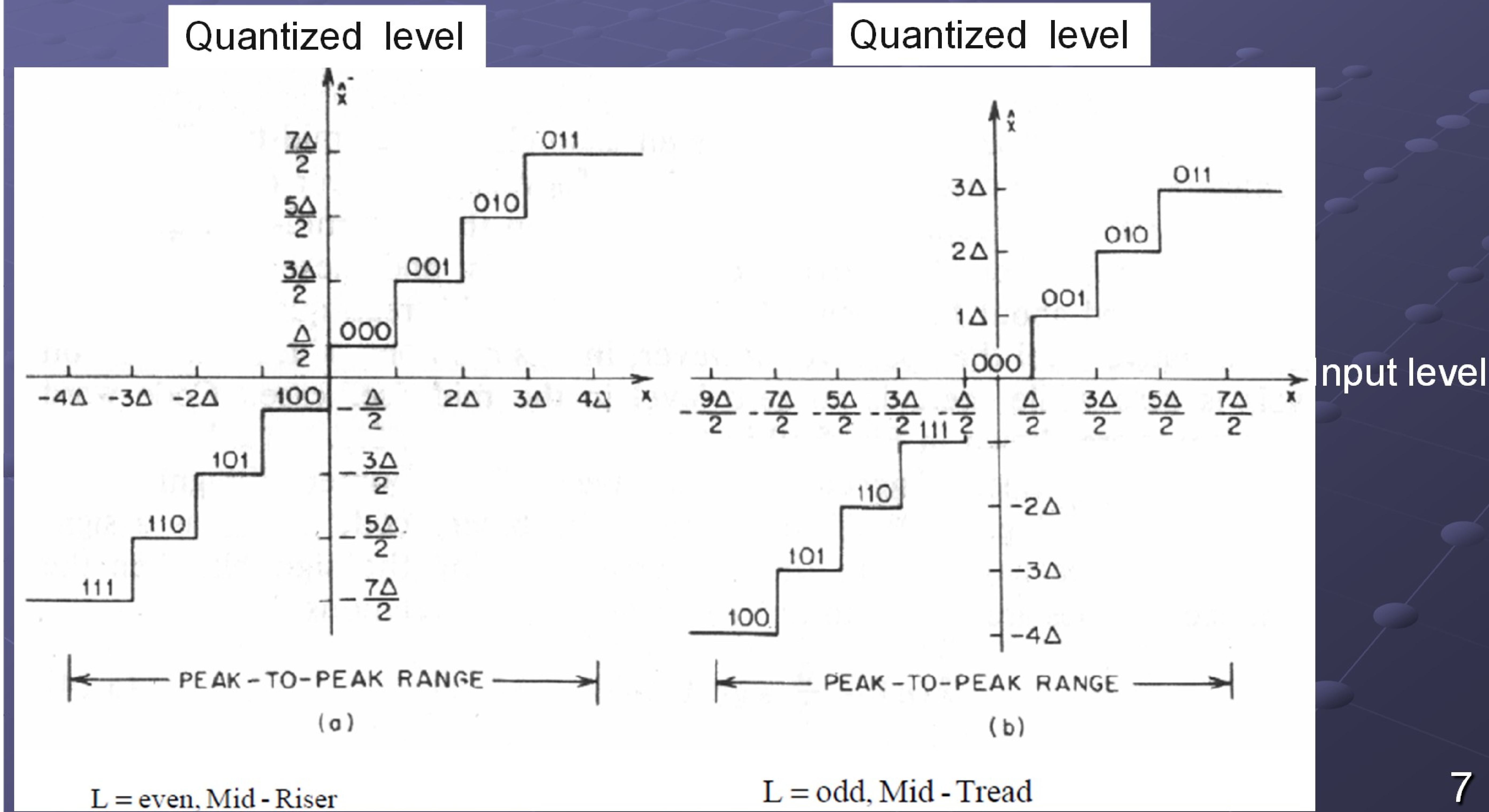
Uniform and non uniform quantization

- Quantizers can be classifieds into uniform or non uniform
- In uniform quantizers the step size is equally spaced between the adjacent levels
- In non uniform step size Δ is made variable according to the sample amplitude

Uniform quantization

- simplest
- Most popular
- Conceptually of great importance
- The input output characteristics of the quantizer are
  1. Staircase-like
  2. Non-linear
- Qunatizers can be classifieds into midtread or midrise

Midtread and midrise quantizers



Quantization noise

- The use of quantization introduces an error between the input signal m and the output signal v known as the quantization noise
- The quantization error is defined as $q = m - v$
- The value of the quantization noise will be $\pm\frac{\Delta}{2}$, where Δ is the step size

pdf of the quantization noise

- Usually the quantizer input m is a sample value of zero-mean random variable M
- This means that the quantization noise is a random variable which can be expressed as $Q = M - V$
- Now we need to find and expression for the signal to quantization noise ratio

Signal to quantization noise ratio

- Consider the input signal m of continuous amplitude in the range $(-m_{max}, m_{max})$,
- The step size Δ will be defined as $\Delta = \frac{2m_{max}}{L} = \frac{m_{pp}}{L}$, where $L\ is\ the\ total\ number\ of levels$

Signal to quantization noise ratio

- If the number of quantization levels L is sufficiently large (Δ is small), we can assume that the quantization error Q is uniformly distributed
- This means that the pdf of Q can be written as

$$f_Q(q) = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} \leq q \leq \frac{\Delta}{2} \\ 0 & otherwise \end{cases}$$

Signal to quantization noise ratio

- With the mean of the quantization error being zero, its variance

$$\sigma^2{}_Q = E[Q^2]$$

$$= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 f_Q(q) dq$$

$$\sigma^2{}_Q = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq = \frac{\Delta^2}{12}$$

Note that the variance represents the average ac power contained in the noise

Signal to quantization noise ratio

- Recall that $\Delta = \frac{2m_{max}}{L}$
- For binary representation the number of levels can be written as

$L = 2^R, where\ R\ is\ the\ total\ number\ of\ bits$

- By substituting the values of L and Δ in $\sigma^2{}_Q$ we have

$$\sigma^2{}_Q = \frac{1}{3}\frac{m^2{}_{max}}{2^{2R}}$$

Signal to quantization noise ratio

- If the average power of the message signal $m(t)$ is denoted by P, then the signal to noise ratio at the output of the quantizer

$$(SNR)_0 = \left(\frac{3P}{m^2{}_{max}}\right)2^{2R}$$

- Note that the signal to noise ratio at the output of the quantizer can be increased by increasing the number of bits or number of levels

Standard number of bits for audio A/D

- Standard audio A/D converters uses 8 bits, while 16 bits are used in sound cards in PC system

Example 1

- A given A/D converter is excited by a sine wave whose peak amplitude is A_m, determine the signal to noise ratio in dB if 5 bits are used in the quantizer, repeat the problem if 8 bits are used

Solution

- The average power in a sinusoidal wave is given by
- $P_{av} = \frac{A^2 m}{2}$
- The signal to noise ration can be found from the expression $(SNR)_O = \left(\frac{3P}{m^2{}_{max}}\right)2^{2R}$

as

$$(SNR)_O = \left(\frac{3\frac{A^2 m}{2}}{A^2 m}\right)2^{2R} = \frac{3}{2}2^{2R}$$

Solution

- Converting the previous expression to dB, we can write

$$10log(SNR)_O = 1.8 + 6R$$

For $\mathrm{R} = 5, (SNR)O = 31.8\ dB$

For $\mathrm{R} = 8, (SNR)_O = 49.8\ dB$

Example 2

- The information in analog waveform, with a maximum frequency $f_m = 3\ kHz$, is to be transmitted over an *M*-ary PAM system, where the number of PAM levels is $L = 16$. The quantization distortion is specified not to exceed $\pm 1\%$ of the peak to peak analog signal

Example 2 Relation between the number of bits and the noise level

a) What is the minimum number of bits/sample, or bits/PCM that should be used in digitizing the analog waveform

b) What is the minimum required sampling rate, and what is the resulting bit transmission rate

c) What is the PAM pulse or symbol transmission rate?

d) If the transmission bandwidth (including filtering) equals 12 kHz, determine the bandwidth efficiency of this system

Solution example 2

a) Note that the maximum quantization error does not exceed $q \leq \frac{\Delta}{2}$. It is also defined for this example as $q = 0.01 m_{pp}$. Equating both equations

$$\frac{\Delta}{2} = 0.01 m_{pp}$$

$$\frac{1}{2}\frac{m_{pp}}{2^R} = 0.01 m_{pp}$$

$$\Rightarrow$$

$$R = \log_2 \frac{1}{2 \times 0.01} = 5.6\ bits$$

$$or\ R \approx 6\ bits$$

Solution example 2

b) According to the Nyquist criterion, the sampling rate would be $f_s = 2f_m = 6000\ smples/second$. The bit transmission rate would be the number of bits multiplied by the sampling rate $R_b = R \times f_s = 6 \times 6000 = 36000\ bit/second$

Solution example 2

c) Since multilevel pulses are to be used with $l = 2^R = 16 \rightarrow R = 4\ bits/symbol$. Therefore the bit stream will be partitioned into groups of 4 bits to form the new 16-level PAM digits, and the resulting transmission rate is $R_{PAM} = \frac{R_b}{4} = \frac{36000}{4} = 9000\ symbols/second$

Solution example 2

d) The bandwidth efficiency is defined as the data throughput per hertz,

$$R_b \frac{R_b}{channel\ BW} = \frac{36000}{12000} = 3 \frac{bits}{s} / Hz$$

non uniform quantization

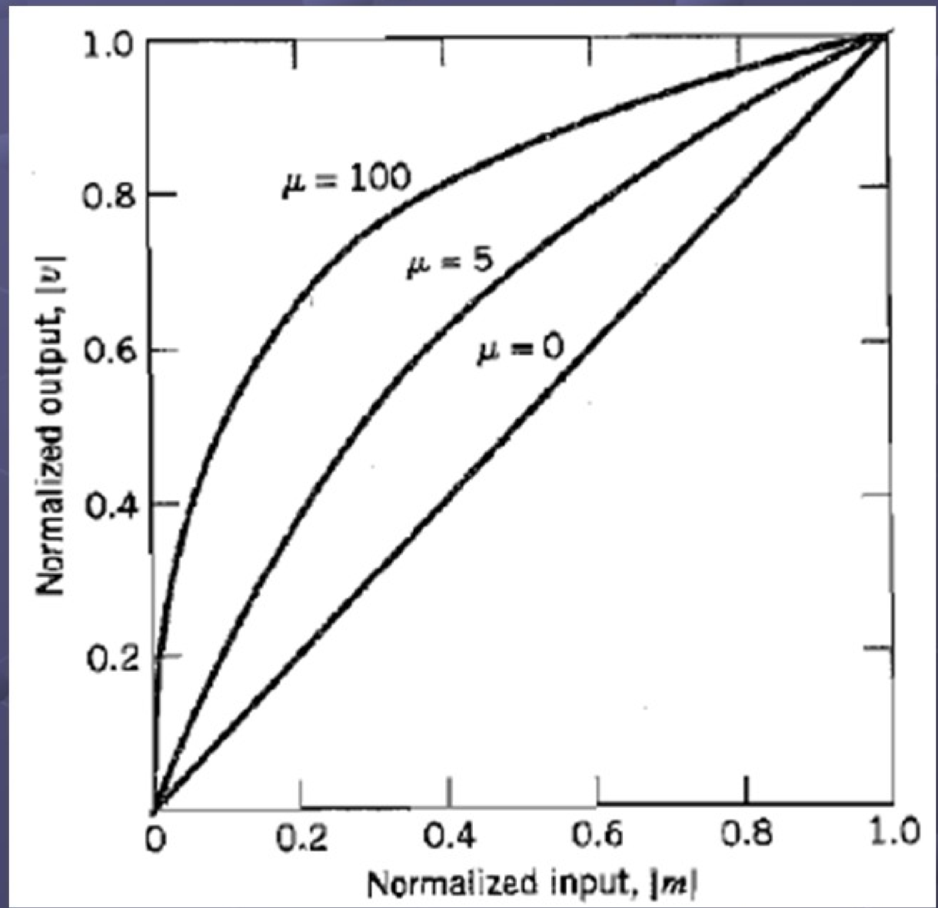
- Non uniform quantization is used in telephophonic communications
- Analysis shows that the ratio of the peaks of the loud talks to the peaks of the weak talks is in the order of 1000 to 1
- In non uniform step size Δ is made variable according to the sample amplitude

non uniform quantization

- The use of a non uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the signal to a uniform quantizer
- Two commonly used compression laws known as $\mu - law\ and\ A - law$ can be applied to the signal to achieve compression

$\mu - law$ compression law

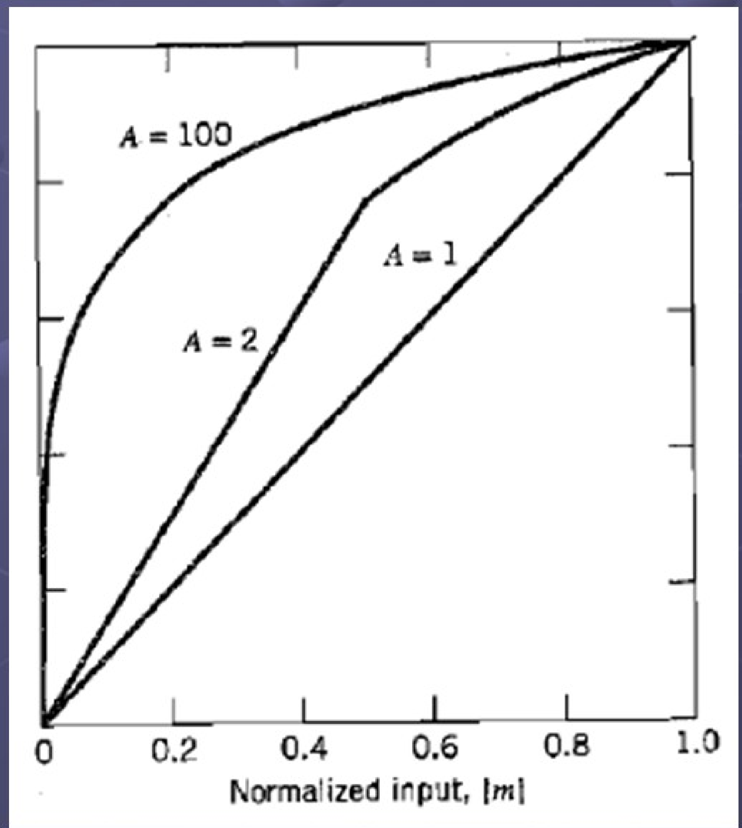
- The $\mu - law$ is defined by $|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$ where m and v are the normalized input and output of voltages, μ is a positive integer



$A - law$ compression law

- The $A - law$ is defined by

$$|v| = \begin{cases} \frac{A|m|}{1+\log A}, & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\log(A|m|)}{1+\log A}, & \frac{1}{A} \le |m| \le 1 \end{cases}$$



Non uniform quantization

- To restore the signal samples to their correct relative level in the receiver, an expander is used
- The combination of a compressor and an expander is called compander
- For both $\mu - law$ and $A - law$, the dynamic range capability of the compander improves with increasing μ and A

Non uniform quantization

- The SNR for low level signals increases on the expenese of the SNR for high-level signals
- A compromise is usually made in choosing the value of μ and A
- Typical values of $\mu = 255\ and\ A = 87.6$ are used

Pulse Code Modulation PCM

- PCM is the most basic form of digital pulse modulation
- PCM is accomplished by representing the signal in discrete form in both time and a amplitude
- PCM can be described by the block diagram shown in the next slide

PCM block diagram

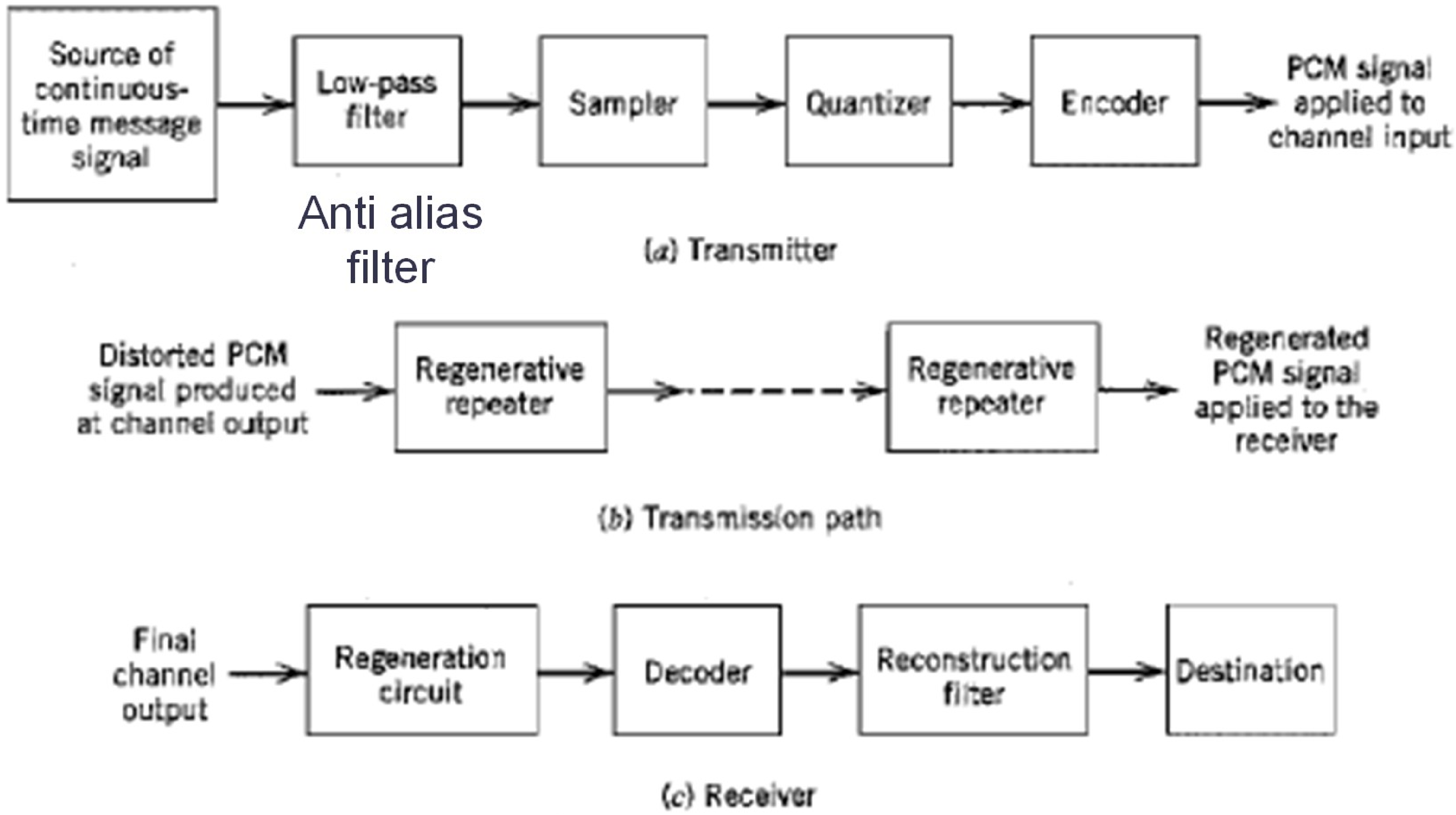


FIGURE 3.13 The basic elements of a PCM system.