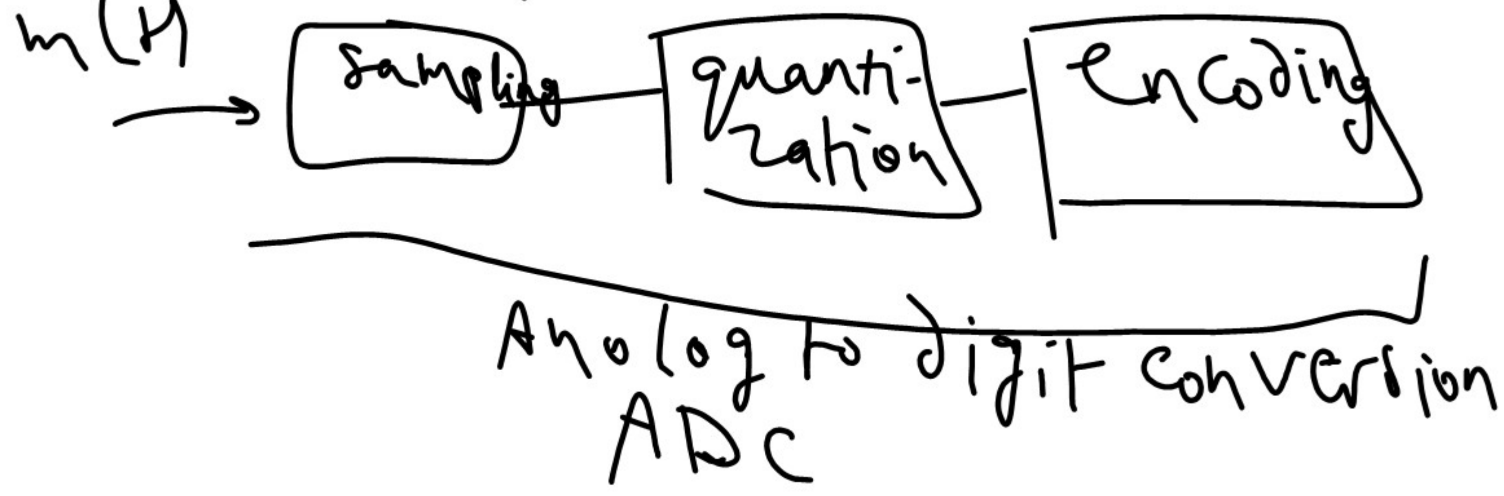
Quantization

* Quantization is the process of Converting discrete-time Continuous amplitude signal discrete-time discrete amplitude Signal
* quantization is used either in Communiction or in processing Signals via digital computer
* quantizer are followed by an encoder
* The signal that results at the encoder output is kwon as Pulse code modulation PCM
* The following three operations



Quantizers can be classifieds into uniform and nonuniform

Uniform quantization

* in uniform quantizers the decision between two levels wich is called the step size Δ is the same between all levels

$$\Delta = \frac{V_p + V_p}{2^R} = \frac{2V_p}{2^R}$$

V_p is the peak amplitude of the signal to be quantized

R is the number of bits

Non uniform quantizers

* In non uniform quantizers, the step size is made variable between the differen levels

* Nonuniform quantization can be used to represent voice signals because, it was found by reseach that human voice contains samples with very magnitude and samples with very large magnitude

* In order to represent the samples with low magnitude we need small $\Delta \rightarrow$ large R

* on the other hand if we made Δ large to represent the samples with large magnitude the the samples with low magnitude will not be presented properly

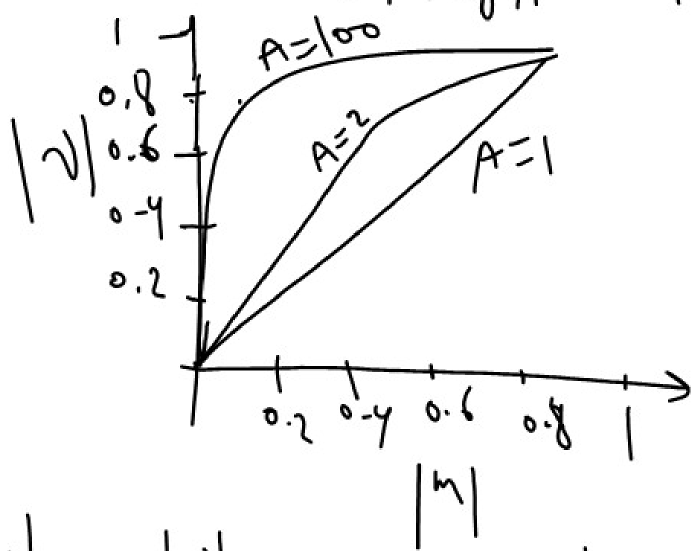
* For the above mentioned reasons we use non-uniform quantization

* Non uniform quantization can be implemented by passing the voice signal through a compressor followed by a uniform quantizer

* There are two commonly used compression laws. These laws are A-law and μ-law

* The A-law is defined mathematically by

$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A} & 0 < |m| \le \frac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A} & \frac{1}{A} < |m| \le 1 \end{cases}$$



where $|v|$ is the magnitude of the sample at the compressor output

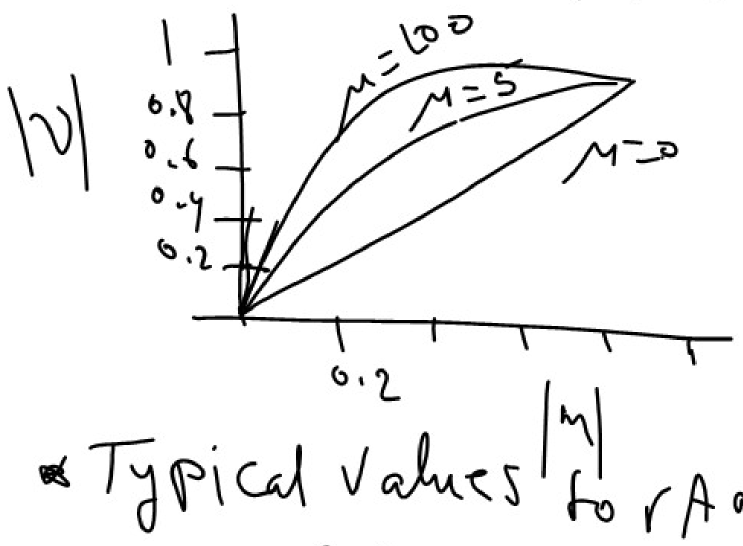
$|m|$ is the magnitude of the input sample

μ- Compression law

* The μ compression law is defined by

$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

If we plot $|v|$ vs $|m|$, then



* Typical values for A and μ are $A = 87.6$, $\mu = 255$

* In order to recover $m(t)$ properly at the receiving end we should use a device called expander

* The use of compressor and expander is abbriviated by compander

Signal to quantization noise ratio

* In this section it is desired to find an expression for signal to quantization noise power
* Note that quantization noise can be classified as a random variable with zero mean and uniform cumulative distribution function cdf
* The noise average power is defined by

$$S_Q = \xrightarrow{\text{zero}}\cancel{\text{DC value}} + \text{AC value}$$

The AC power of the noise is its variance σ_Q^2

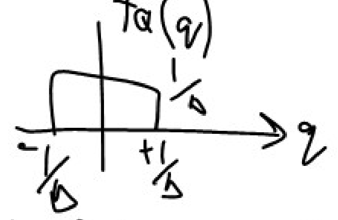
$$\sigma_Q^2 = E[Q^2] = \int_{-\infty}^{\infty} q^2 f_Q(q)\, dq$$

we $f_Q(q)$ is the CDF

and $-\frac{\Delta}{2} < q < \Delta/2$, q is the quantization noise

Note that the CDF for the quantization noise takes a uniform distribution as illustrated by

$$f_Q(q) = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} < q < \Delta/2 \end{cases}$$



$$\sigma_Q^2 = \int_{q=-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta} q^2\, dq = \frac{\Delta^2}{12} \text{ watt}$$

Recall that $\Delta = \frac{2V_p}{L} = \frac{2V_p}{2^R}$

where V_p is the peak amplitude of the message signal

R is the number of quantization bits

$$(SNR)_Q = \frac{S}{\sigma_Q^2} = \frac{S}{\Delta^2/12} = \frac{12S}{\Delta^2}$$

$$= \frac{12\,S}{\left(\frac{2V_p}{2^R}\right)^2} = \frac{3\,S\,2^{2R}}{V_p^2}$$

Ex

A given A/D converter is tested by $m(t) = A_m \cos \omega t$

a) Determine an expression for $(SNR)_Q$ in terms of R

b) Determine $(SNR)_Q$ in dB If $R = 5$, $R = 8$ bits

Solution

a) $$(SNR)_Q = \frac{3 S 2^{2R}}{V_p^2}$$

Note that $S = \frac{A_m^2}{2}$, $V_p = A_m$

$$(SNR)_Q = \frac{3 \frac{A_m^2}{2} 2^{2R}}{A_m^2}$$

$$= \frac{3}{2} 2^{2R}$$

$$(SNR)_Q = 10 \log_{10} \frac{3}{2} + 10 \log_{10} 2^{2R}$$

$$= 1.8 + 6R \text{ dB}$$

For $R = 5 \rightarrow (SNR)_Q = 1.8 + 6(5)$

$= 31.8$ dB

For $R = 8 \rightarrow (SNR)_Q = 1.8 + 6(8) = 49.8$ dB

the standard number of quantization bits is $R = 8$ for telephone communications

$R = 16$ bits for CD recordings

The standard sampling rates are

$f_s = 8$ kHz for telephone comm

$f_s = 44.1$ kHz for CD recordings

Note that the Bandwidth required for the transmission of PCM signal is $BW = R f_s$

Ex The information in analog waveform with a maximum frequency $f_m = 3\,kHz$ is to be transmitted as a PCM signal.
The quantization noise should not exceed $\pm 1\% V_{pp}$

a) Determine the minimum number of bits such that the quantization noise does not exceed $\pm 1\% V_{pp}$

b) Determine the minimum sampling rate. What is R_b

c) Sketch the block diagram of the PCM system

d) If the available channel bandwidth is 12 kHz, design a PAM system that can be used to transmit the signal without distortion

e) Determine the bandwidth efficiency

Solution

a) $q = \frac{\Delta}{2} = 0.01 V_{pp}$

$$= \frac{\cancel{2V_p}}{2\, 2^R} = \cancel{0.01 V_{pp}}$$

$$2^R = \frac{1}{2 \times .01} = 50$$

$R = 5.6 \approx 6$ bits

b) $f_s = 2 f_m$

$= 2(3000) = 6000\,Hz$

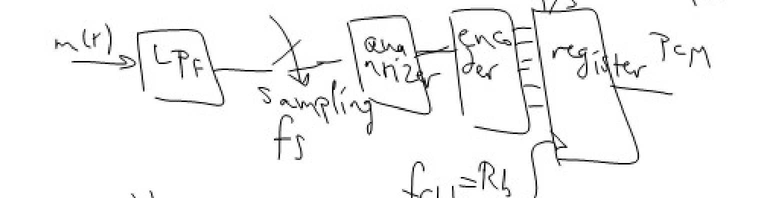
$R_b = R \times f_s$

$= 6 \times 6000 = 36\,kbits/s$

Note that the bandwidth required for the transmission of a PCM is

$BW = R_b = 36\,kbits/s$

c) The block diagram of the PCM would be as shown below



d) Since the PCM data bandwidth is greater than the channel bandwidth we use a multilevel encoding M-ary transmission

* In M-ary encoding the binary data are transmitted as a group of PAM symbols
The symbol rate (PAM)

$$R_s = \frac{R_b}{\log_2 M}$$

where $\log_2 M$ is the number of bits to generate M symbols in M-ary system

If we use M-ary transmission, then the symbols bandwidth

$BW = R_s = 12\,kHz$ (symbol/s)

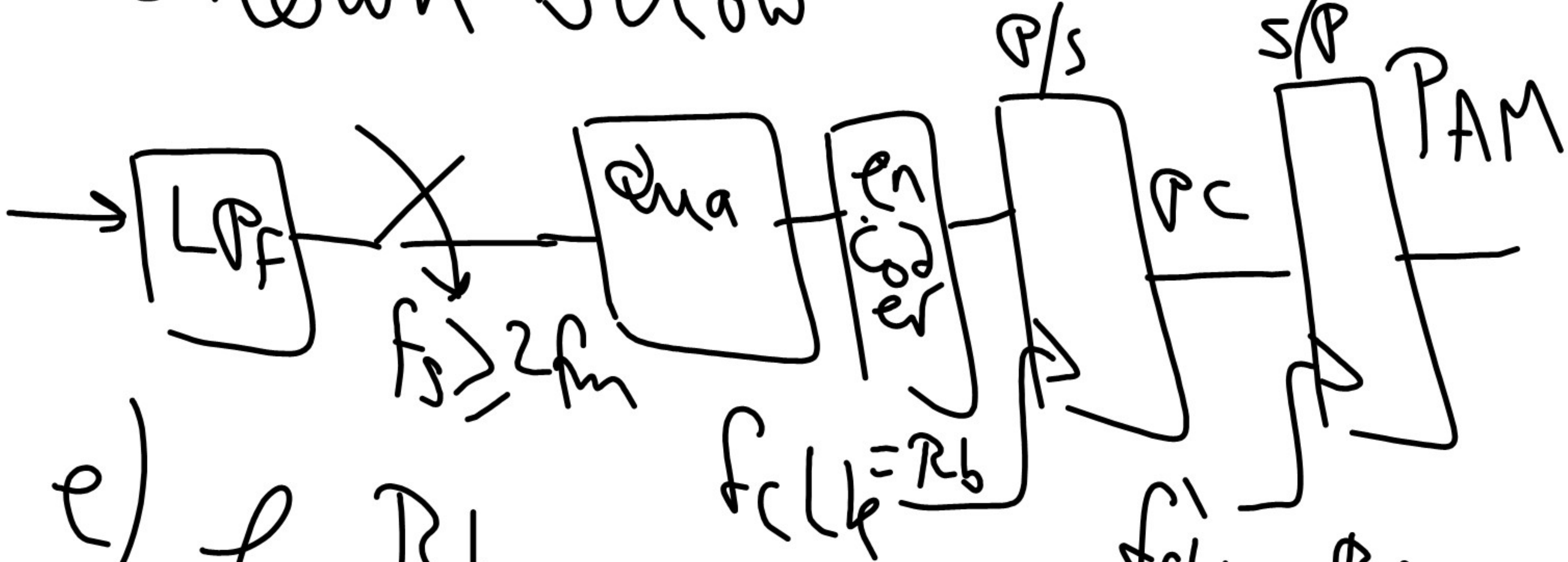
$\therefore R_s = \frac{R_b}{R}$

Therefore

$$R = \frac{R_b}{R_s} = \frac{36000}{12000} = 3 \text{ bits}$$

$$\text{or } M = 2^3 = 8\text{-levels}$$

The block diagram of the communication system can be modified as shown below



e) $$\rho = \frac{R_b}{\text{channel BW}} = \frac{36 \text{ kbit/s}}{12 \text{ k/s}} = 3 \text{ bits/s/Hz}$$