Correlative level coding is a method to add ISI in the transmitted bits in a controlled manner. By adding this controlled ISI, it would be possible to achieve a signaling rate of $2W$ symbols per second in a channel of bandwidth of $W$ Hz.
Duo binary signaling

The idea of correlative level coding is illustrated by considering the duo binary signaling.

“duo” means doubling of the transmission capacity of a straight binary system.
Duo binary signaling (continued)

Consider a binary sequence \( \{ b_k \} \) consisting of uncorrelated binary symbols 1 and 0, each having a duration \( T_b \).

This sequence is applied to a pulse amplitude modulator producing two-level sequence of short pulses:

\[
a_k = \begin{cases} 
+1 & \text{if symbol } b_k \text{ is 1} \\
-1 & \text{if symbol } b_k \text{ is 0}
\end{cases}
\]
Duo binary encoder

The duo binary encoder converts the two levels binary sequence

\[ a_k = \begin{cases} 
+1 & \text{if symbol } b_k \text{ is 1} \\
-1 & \text{if symbol } b_k \text{ is 0}
\end{cases} \]

into three level output, namely, -2, 0, +2

This transformation can be achieved by using the equation \( c_k = a_k + a_{k-1} \) which is implemented as shown in the next slide.
Duo binary encoder

Block diagram of the duo binary encoder
Physical meaning of duo binary level coding

The transformation described by
\[ c_k = a_k + a_{k-1} \]
changes the input two level sequence of uncorrelated binary sequence \( \{a_k\} \) into a sequence of correlated three levels pulses \( \{c_k\} \)

This correlation between adjacent pulses may be viewed as introducing ISI in the transmitted signal in an artificial manner.
Impulse response of the duo binary encoder

The impulse response of the duo binary encoder can be derived as follows

\[ H_I(f) = H_{Nyquist}(f) \left[ 1 + e^{-j2\pi f T_b} \right] \]

\[ H_I(f) = H_{Nyquist}(f) \left[ e^{j\pi f T_b} + e^{-j\pi f T_b} \right] e^{-j\pi f T_b} \]

\[ H_I(f) = 2H_{Nyquist}(f) \cos(\pi f T_b) e^{-j\pi f T_b} \]
Impulse response of the duo binary encoder

Recall from the ISI section that the frequency response of an ideal Nyquist channel was defined by

\[ H_{Nyquist}(f) = \begin{cases} 
1 & -\frac{1}{2T_b} \leq f \leq \frac{1}{2T_b} \\
0 & \text{otherwise} 
\end{cases} \]

Now the frequency response of the duo binary encoder became

\[ H_I(f) = \begin{cases} 
2\cos(2\pi f T_b)e^{-j2\pi f T_b} & -\frac{1}{2T_b} \leq f \leq \frac{1}{2T_b} \\
0 & \text{otherwise} 
\end{cases} \]
Graphical representation of $H_r(f)$
The impulse response of the duo binary encoder $H_r(f)$

\[ h_1(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi (t - T_b)/T_b]}{\pi (t - T_b)/T_b} \]

\[ = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi (t - T_b)/T_b} \]

\[ = \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \]
Notes about $H_1(f)$

- The response of a pulse is spread over more than one signaling interval
- The response is partial in any signaling interval
Detection (decoding) of duo binary signals

In order to detect or recover the bits into its original format we can reverse the operation made in the modulator according to the following equation:

\[ \hat{a}_k = c_k - \hat{a}_{k-1} \]

Decision feed back
Example of duo binary coding and decoding

Consider the binary sequence $b_k = 0010110$

find the coded sequence $\{c_k\}$ and the decoded sequence $\{\hat{a}_k\}$ assume the first bit is the reference bit

<table>
<thead>
<tr>
<th>$b_k$</th>
<th>0 0 1 0 1 1 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_k$</td>
<td>-1 -1 +1 -1 +1 +1 -1</td>
</tr>
<tr>
<td>$c_k$</td>
<td>-2 0 0 0 2 0</td>
</tr>
<tr>
<td>$\hat{a}_k$</td>
<td>-1</td>
</tr>
<tr>
<td>$\hat{b}_k$</td>
<td>0 1 0 1 1 1 0</td>
</tr>
</tbody>
</table>

$c_k = a_k + a_{k-1}$

$\hat{a}_k = c_k - \hat{a}_{k-1}$
Shortcomings of duo binary coding

The duo binary coding suffers from two shortcomings

1. If one a given bit received in error, then the error tends to propagate in the succeeding bits. This known as error propagation

2. It can be seen from the $|H_I(f)|$ the frequency response contains a DC components which makes it improper for AC communication channels
Error propagation can be solved by using pre-coding.

In pre-coding the binary sequence \( \{b_k\} \) is converted into another binary sequence \( \{d_k\} \) according to

\[
d_k = b_k \oplus d_{k-1}
\]

Where the \( \oplus \) denotes modulo-two addition (XOR)
Block diagram of duo binary encoder with pre-coding
Duo binary sequence $c_k$ with pre-coding notes

The coded sequence $\{c_k\}$ produced with pre-coding consists of three levels as illustrated by $c_k = a_k + a_{k-1}$

$$c_k = \begin{cases} 0 & \text{if data symbol is 1} \\ \pm 2 & \text{if data symbol is 0} \end{cases}$$
Detection of original binary data from $c_k$ pre-coding

It can be noticed that

- if $|c_k| < 1$ symbol $b_k$ is 1
- if $|c_k| > 1$ symbol $b_k$ is 0

From this discussion the decoder can be a simple rectifier followed by a decision device (comparator) as shown in the next slide
Block diagram of the detector with precoding

\[ \{c_k\} \xrightarrow{\text{Rectifier}} \{ |c_k| \} \xrightarrow{\text{Decision device}} \]

- Say \( b_k = 1 \) if \( |c_k| < 1 \)
- Say \( b_k = 0 \) if \( |c_k| > 1 \)

Threshold = 1
Example Duo binary coding with pre-coding

<table>
<thead>
<tr>
<th>Binary sequence ({b_k} )</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precoded sequence ({d_k} )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Two-level sequence ({a_k} )</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>Duobinary coder output ({c_k} )</td>
<td>+2</td>
<td>+2</td>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>Binary sequence obtained by applying decision rule of Eq. (4.76)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Modified duo binary signaling

- In duo binary signaling, $H(f)$ is nonzero at the origin.
- We can correct this deficiency by using the class IV partial response.

**Figure 4.16** Modified duobinary signaling scheme.
Frequency response of the modified duo binary signaling

\[ H_{IV}(f) = H_{Nyquist}(f)[1 - \exp(-j4\pi f T_b)] = 2jH_{Nyquist}(f)\sin(2\pi f T_b) \exp(-j2\pi f T_b) \]

\[ H_{IV}(f) = \begin{cases} 
2j \sin(2\pi f T_b) \exp(-j2\pi f T_b), & |f| \leq 1/2T_b \\
0, & \text{elsewhere} 
\end{cases} \]

**Figure 4.17** Frequency response of the modified duobinary conversion filter. (a) Magnitude response. (b) Phase response.
Impulse response of the modified duo binary signaling scheme

Time Sequency: interpretation of receiving 2, 0, and -2?

\[
    h_{IV}(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t - 2T_b)/T_b]}{\pi(t - 2T_b)/T_b}\]

\[
    = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t - 2T_b)/T_b}
\]

\[
    = 2T_b^2 \frac{\sin(\pi t/T_b)}{\pi t(2T_b - t)}
\]

**Figure 4.18** Impulse response of the modified duobinary conversion filter.