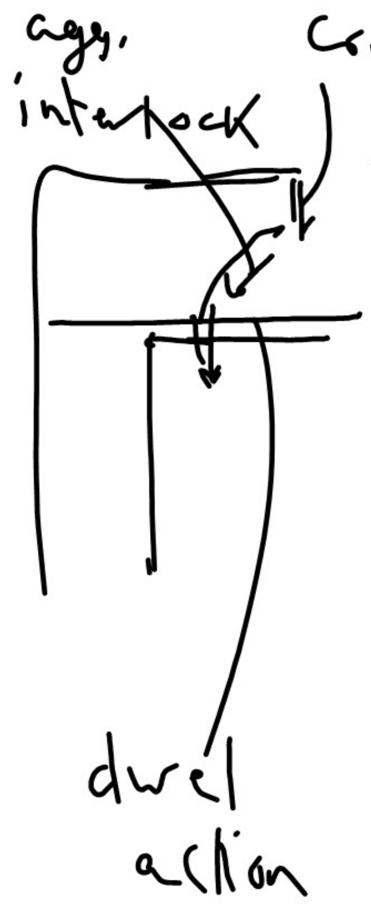
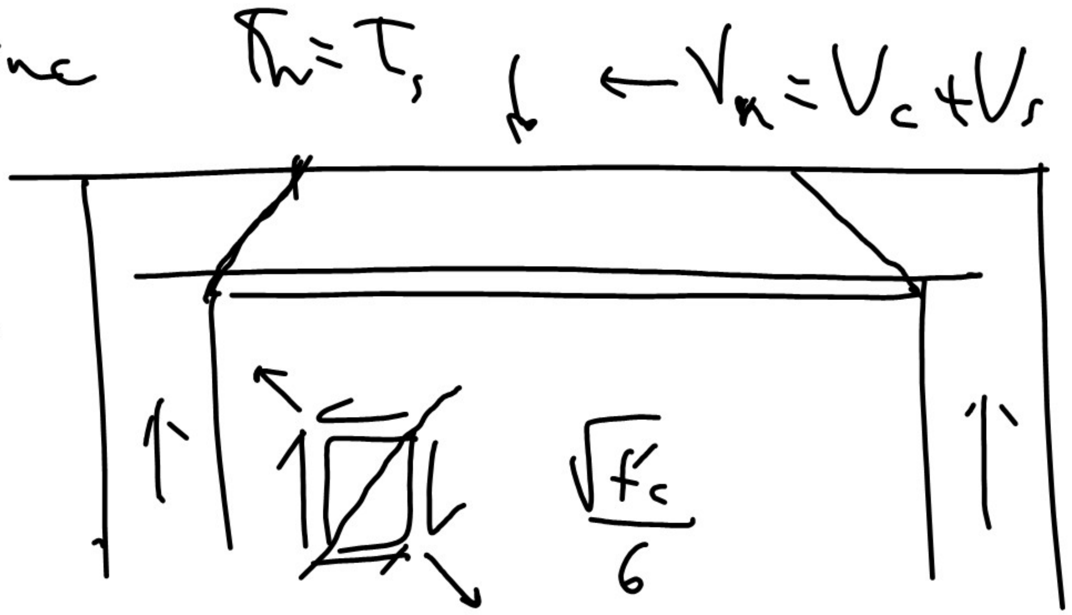
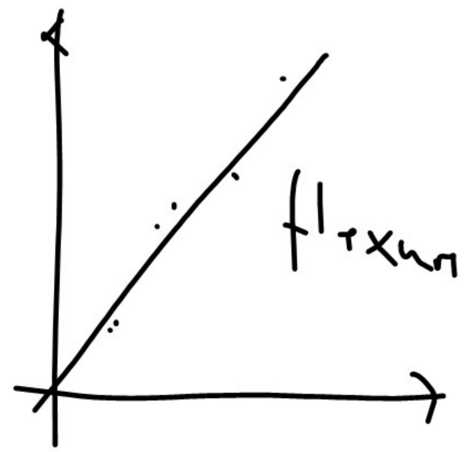
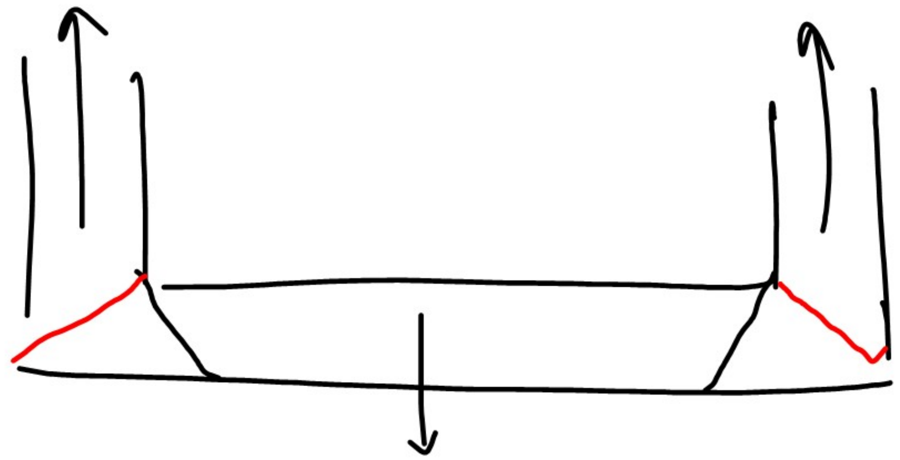
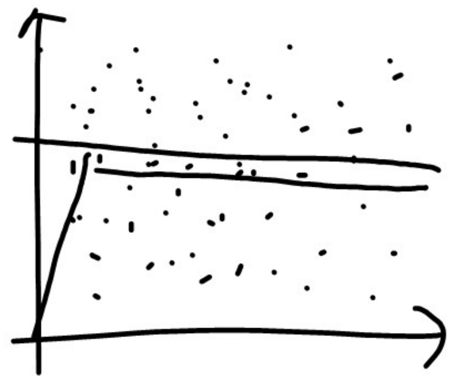

agg.
interlock
crack
dowel
action


$T_n = T_s$
$V_n = V_c + V_s$
$\frac{\sqrt{f'_c}}{6}$


flexure



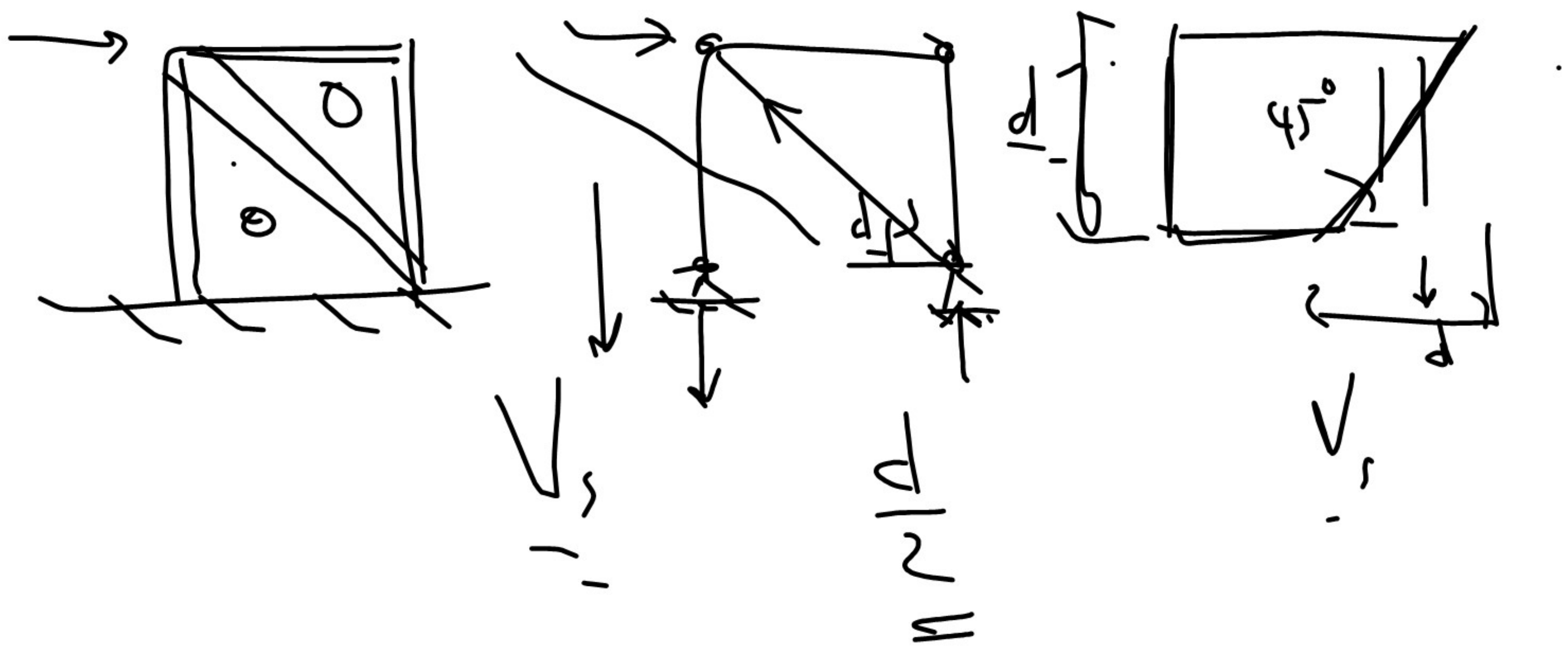


d = 20

10

15

25


d
45°
d
V_s
$\frac{d}{2}$
V_s

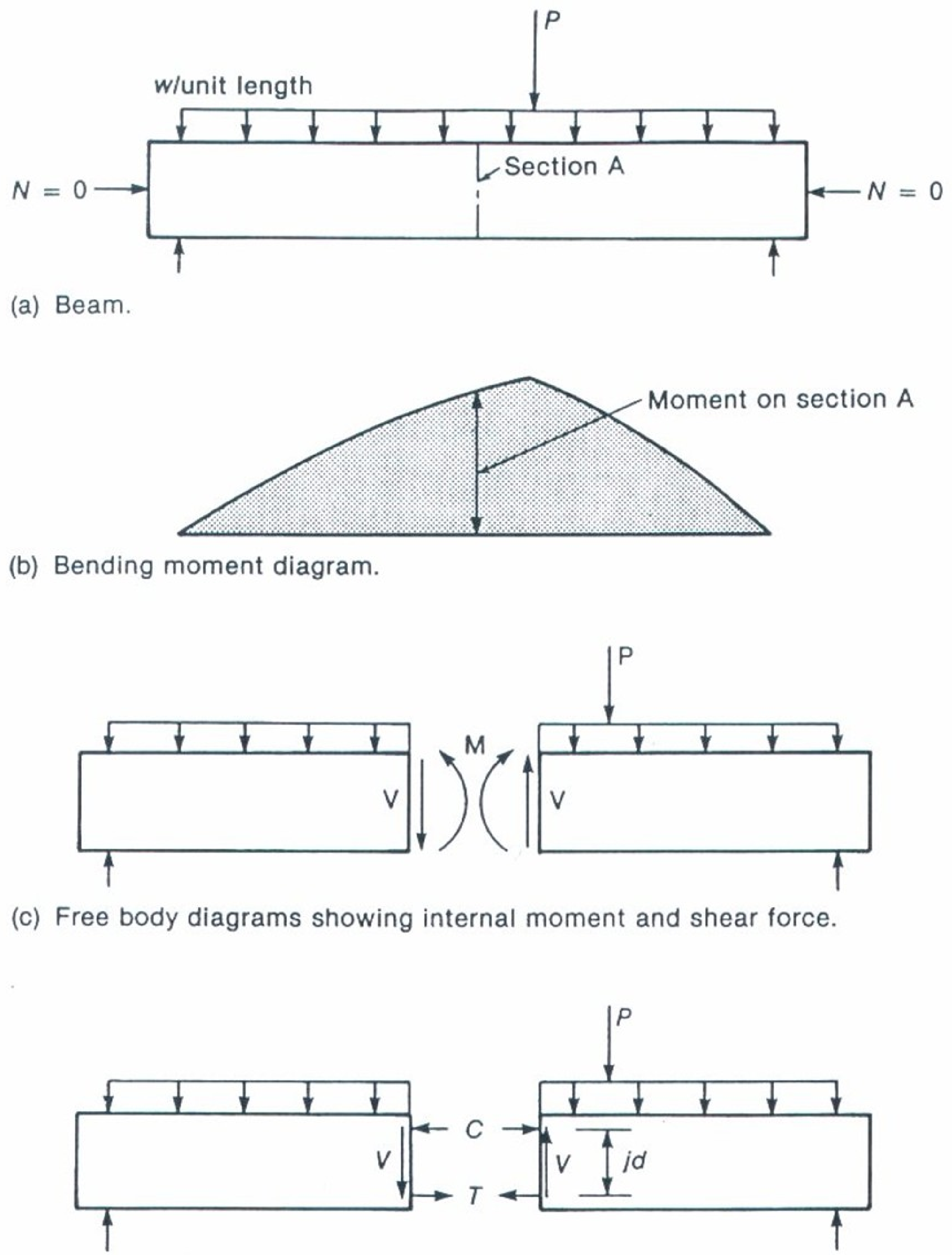
Beams: 4.2.1 Flexure

The beam is a structural member used to support the internal moments and shears. It would be called a beam-column if a compressive force existed.

$$C = T$$

$$M = C*(jd)$$

$$= T*(jd)$$



(a) Beam.

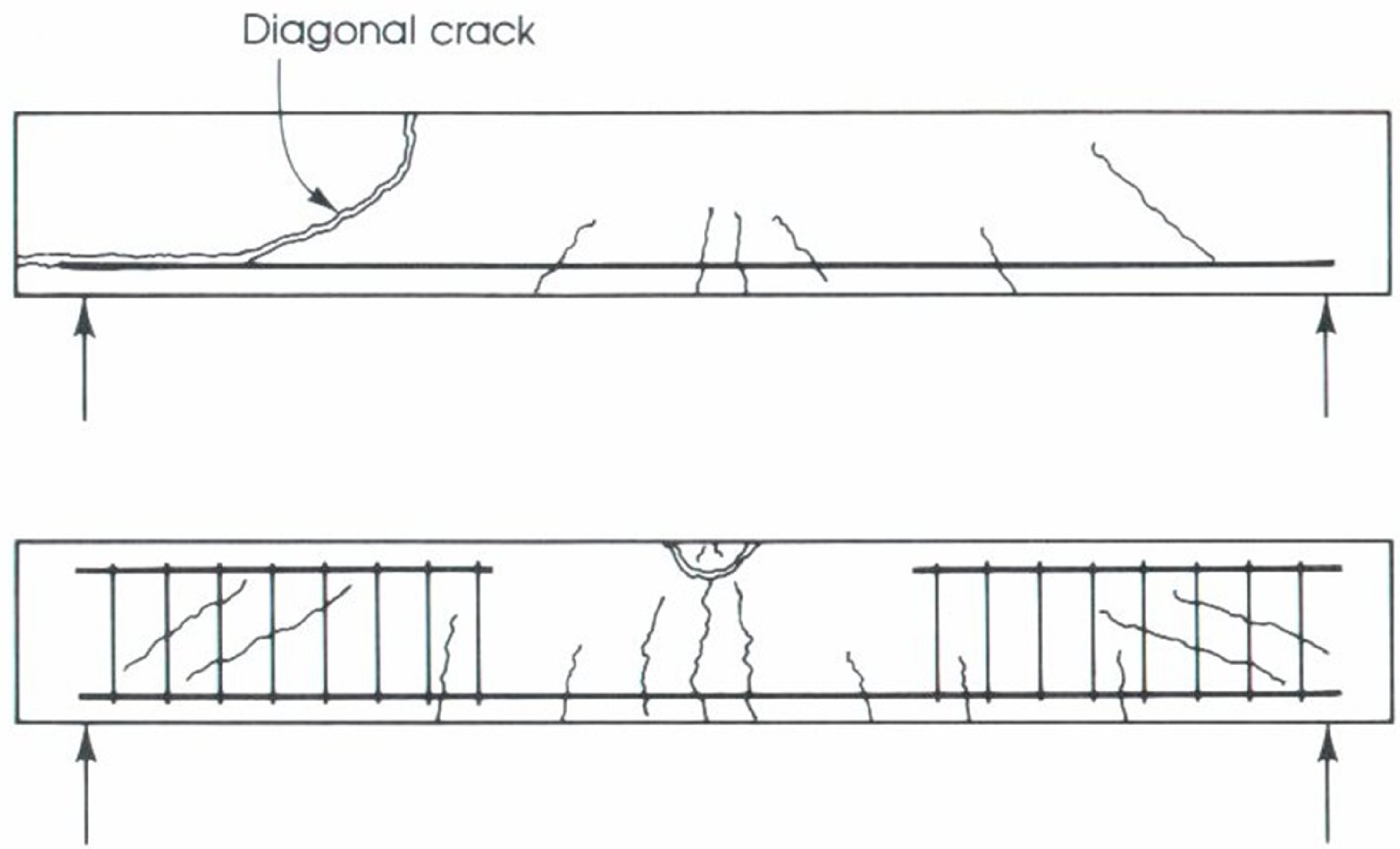
(b) Bending moment diagram.

(c) Free body diagrams showing internal moment and shear force.

(d) Free body diagrams showing internal moment as a compression-tension force couple.

Flexure

The first beam fails in shear, the second fails in bending moment.



Approximate analysis

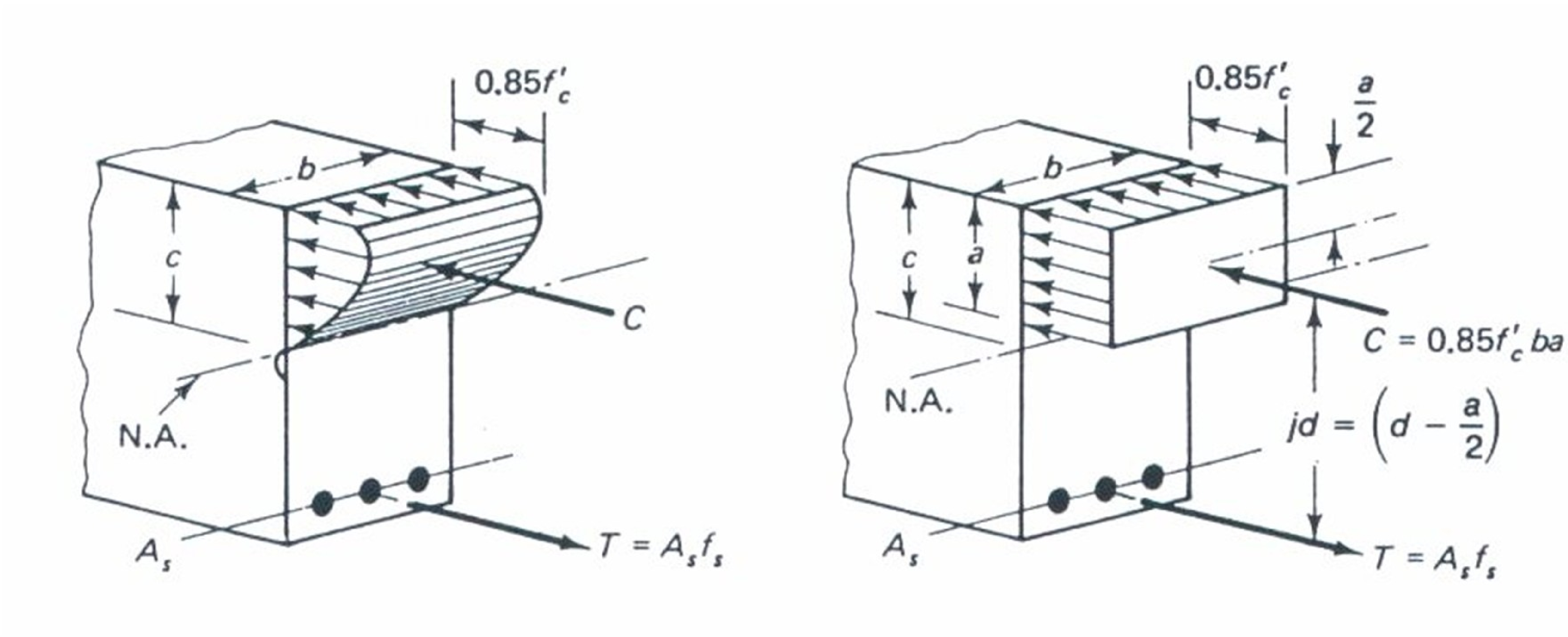
- Use of tributary area (area of floor or roof which supports all of the loads whose load path leads to the beam) determines the beam load.
- Perform approximate analysis through:
  - Approximate deflected shape to locate points of inflection, hence transform to determinate beam and analyze using statics.
  - Use analysis coefficients (e.g. ACI coefficients)
  - Use finite element programs

Basic Assumptions in Flexure Theory

- Plane sections remain plane (not true for deep beams h > 4b)
- The strain in the reinforcement is equal to the strain in the concrete at the same level, i.e. $\varepsilon_s = \varepsilon_c$.
- Stress in concrete & reinforcement may be calculated from the strains using f-ε curves for concrete & steel.
- Tensile strength of concrete is neglected.
- Concrete is assumed to fail in compression, when $\varepsilon_c = 0.003$
- Compressive f-ε relationship for concrete may be assumed to be any shape that results in an acceptable prediction of strength.

Design for Flexure: review

The compressive zone is modeled with an equivalent stress block.



Design for Flexure: review

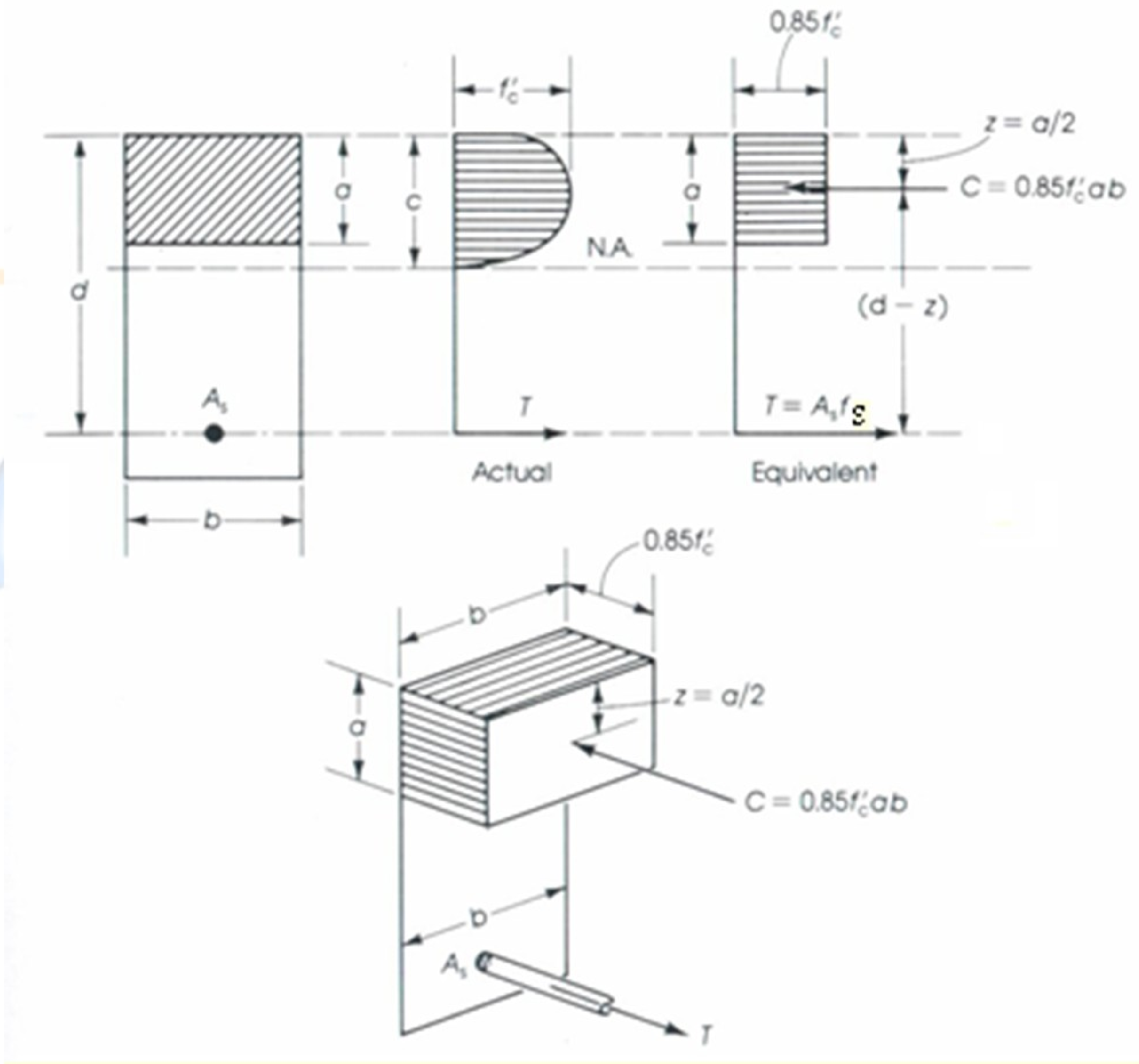
Example of rectangular reinforced concrete beam.

Setup equilibrium.

$$\sum F_x = 0 \quad \Rightarrow \quad \mathrm{T} = \mathrm{C}$$

$$A_s f_s = 0.85 f_c ab$$

$$\sum M = 0 \quad \Rightarrow \quad \mathrm{T}\left(d - \frac{a}{2}\right) = \mathrm{M_n}$$



Design for Flexure: review

The ultimate load, which is used in the design and analysis of the structural member is:

$$M_u = \phi M_n$$

M_u – Ultimate Moment

M_n – Nominal Moment

Φ – Strength Reduction Factor

The strength reduction factor, Φ, varies depending on the tensile strain in steel in tension. Three possibilities:

Compression Failure - (over-reinforced beam)

Tension Failure - (under-reinforced beam)

Balanced Failure - (balanced reinforcement)

Design for Flexure: review

Which type of failure is the most desirable?

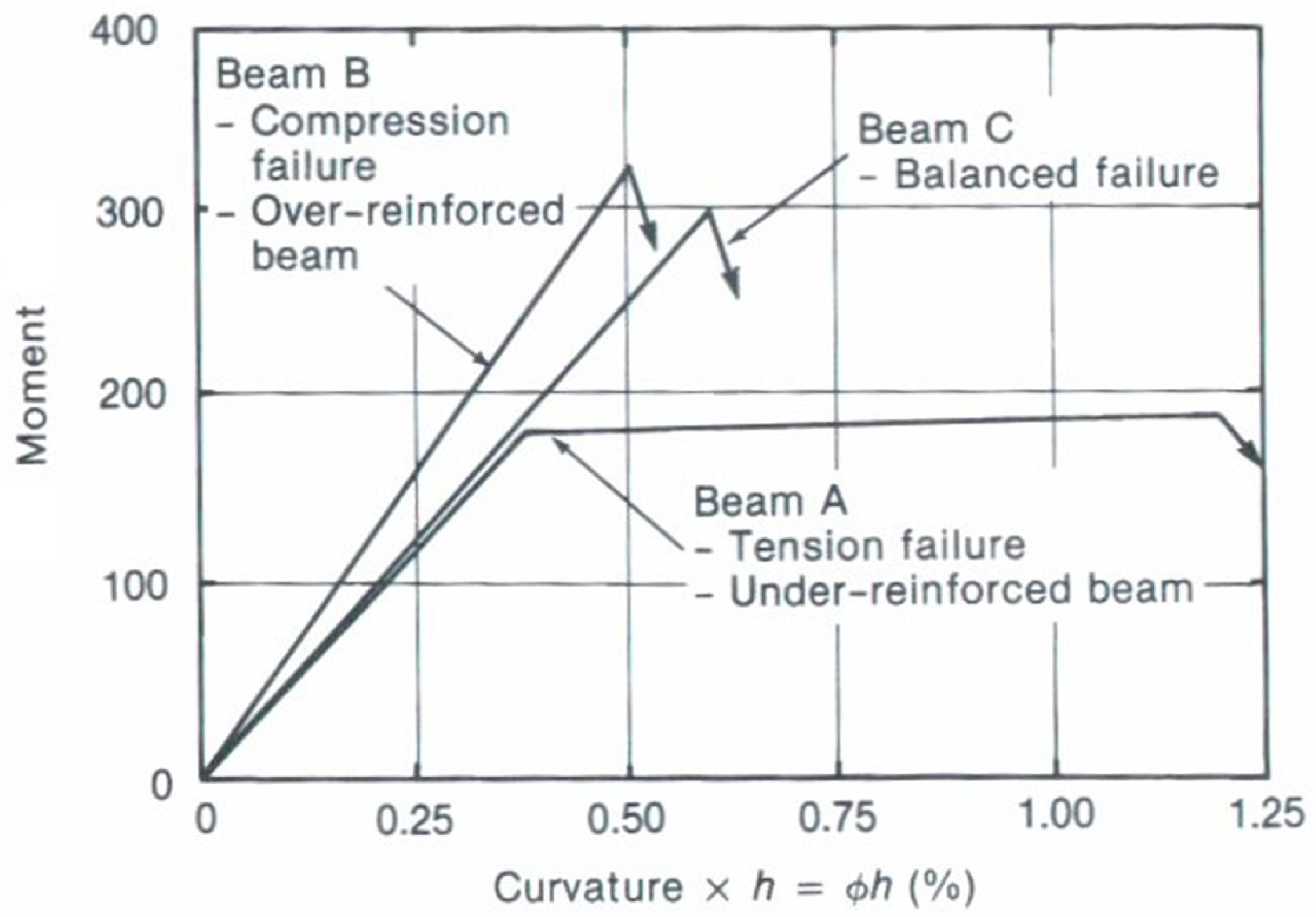
The ***under-reinforced beam*** is the most desirable.

$f_s = f_y$

$\varepsilon_s >> \varepsilon_y$

You want ductility

system deflects and still carries load.



Approximate Design for Flexure: J

For under-reinforced, the equation can be rewritten as:

$$C = T \quad \Rightarrow \quad 0.85 f_c' \mathrm{ba} = \mathrm{A_s} f_y$$

$$\mathrm{a} = \frac{f_y \mathrm{A_s}}{0.85 f_c' \mathrm{b}}$$

$$M_n = \mathrm{A_s} f_y d \left(1 - \frac{f_y \mathrm{A_s}}{1.7 f_c' \mathrm{bd}} \right)$$

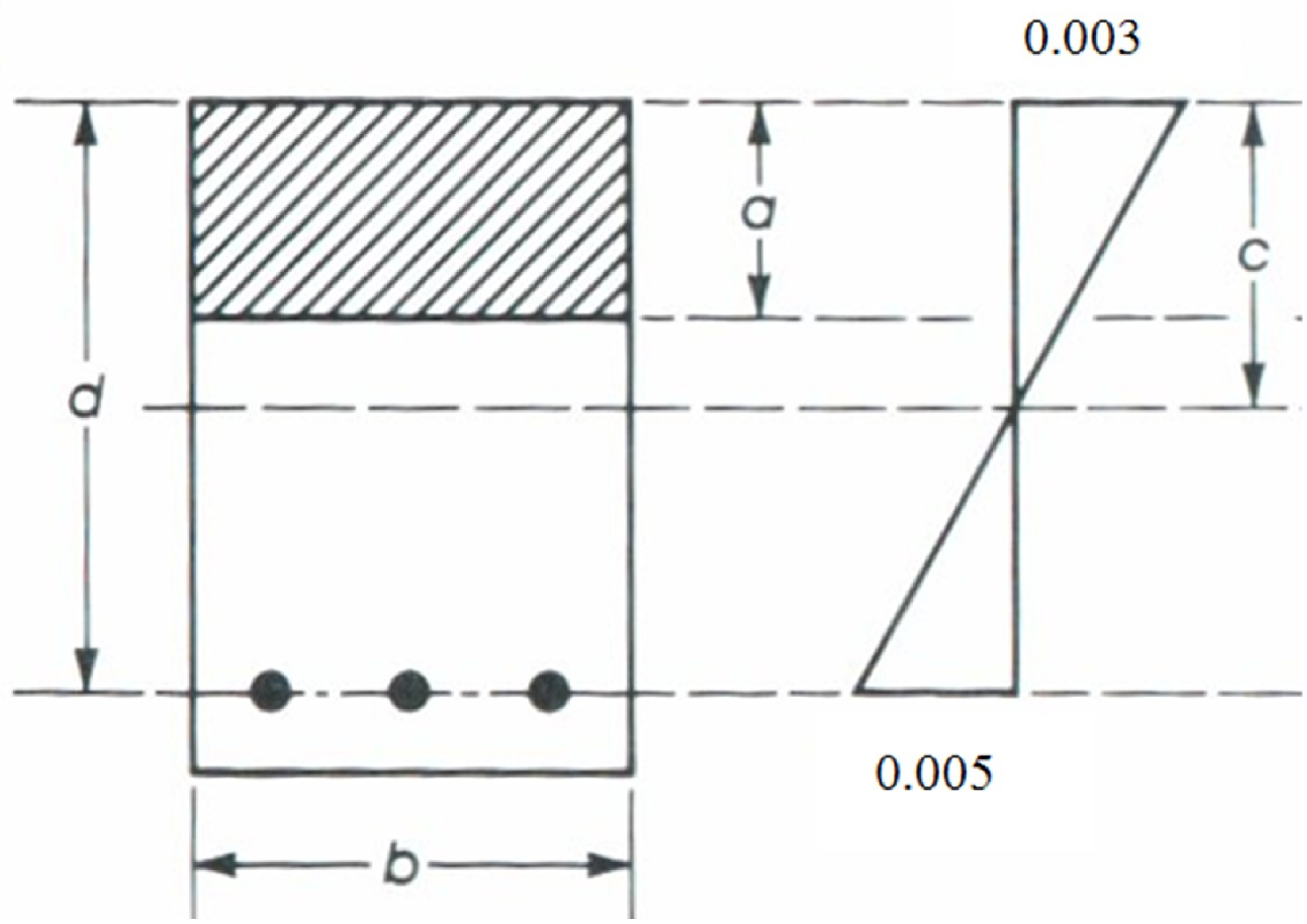
$$M_d = \phi M_n = \mathrm{A_s} d \phi f_y \left(1 - \frac{\rho \mathrm{f_y}}{1.7 f_c'} \right) = \mathrm{A_s} dj$$

Approximate Design for Flexure: J

ρ_{max} = maximum ρ value recommended to get simultaneous $\varepsilon_c = 0.003$ & $\varepsilon_s = 0.005$

Use similar triangles:

$$\frac{0.003}{c} = \frac{0.005}{d - c}$$



Approximate Design for Flexure: J

For a yield stress 420MPa, the equation can be rewritten to find c as

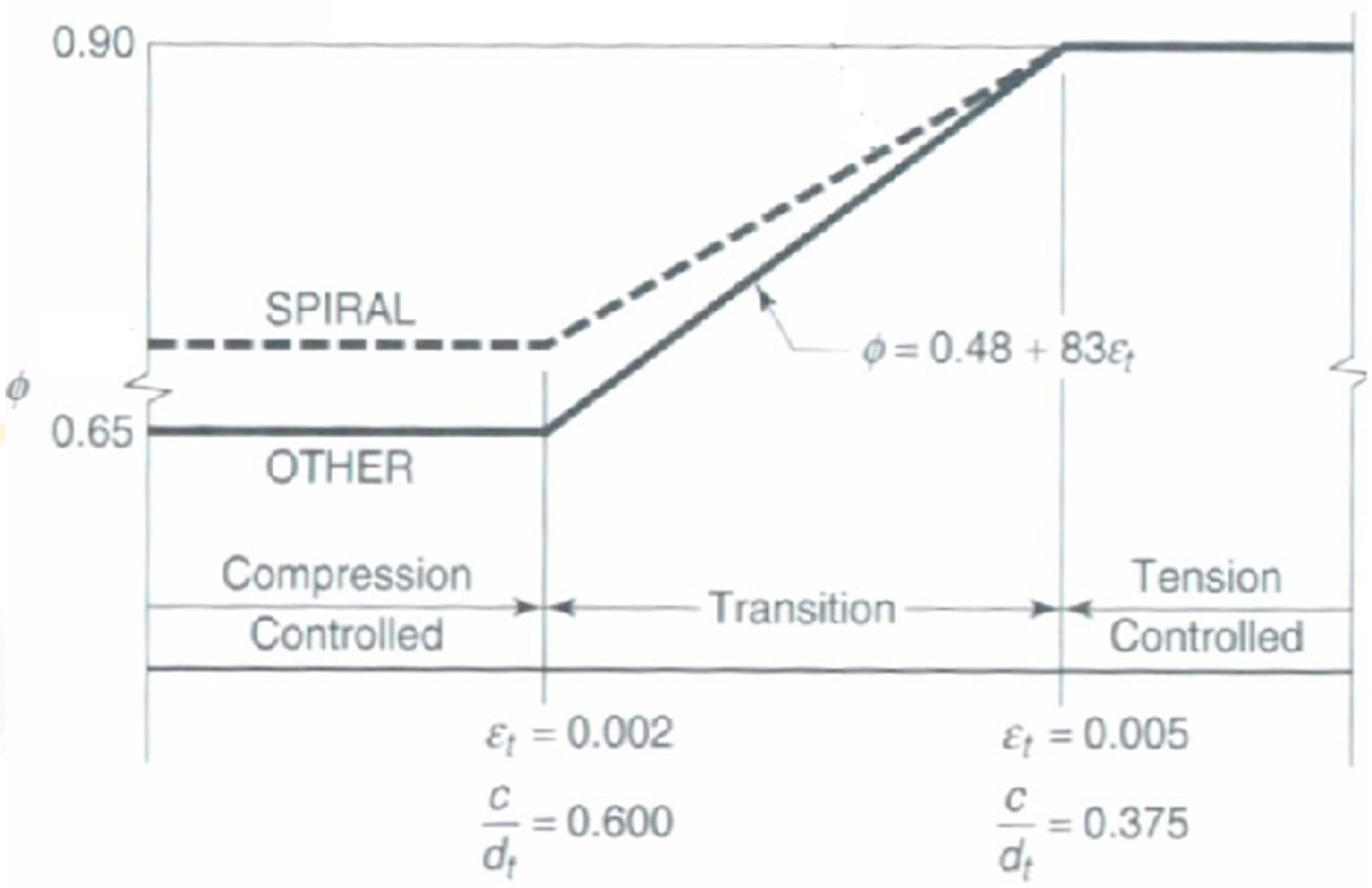
$$c = \frac{0.003d}{0.008} \Rightarrow c = 0.375d$$

$$a = 0.85c = 0.319d$$

$$\rho = 0.271 f_c' / f_y$$

Approximate Design for Flexure: J

The strength reduction factor, ϕ, will come into the calculation of the strength of the beam.



The factor J for large steel ratios

$$M_d = \phi M_n = A_s d \phi f_y \left(1 - \frac{\rho f_y}{1.7 f_c'} \right) = A_s dj$$

$$j = \phi f_y \left(1 - \frac{\rho f_y}{1.7 f_c'} \right)$$

- For concrete strength variation 20MPa to 42Mpa, the value of J for maximum recommended steel ratio varies is 0.317. Moment in kN.m, area of steel in square cm and depth in cm.

Limitations on Reinforcement Ratio, ρ

Lower Limit on ρ ACI 10.5.1

$$A_{s(\min)} = \frac{\sqrt{f'_c}}{4 f_y} * b_w d \geq \frac{1.4}{f_y} * b_w d$$ ACI Eqn. (10-3)

f_c & f_y are in MPa

Lower limit used to avoid "Piano Wire" beams.

Very small As ($M_n < M_{cr}$)

Strain in steel is huge (large deflections)

when beam cracks ($M_u/\Phi > M_{cr}$) beam fails right away because nominal capacity decreases drastically.

The factor J for minimum steel ratios

$$j = \phi f_y \left(1 - \frac{\rho f_y}{1.7 f'_c} \right) = \phi f_y \left(1 - \frac{1.4}{1.7 f'_c} \right)$$

- For concrete strength variation 20MPa to 42Mpa, the value of J for minimum steel varies from 0.36 to 0.37

The factor J

- It is obvious that variation of J is not sensitive to changes in concrete strength. Thus a mean value of 0.33 is representative for all types of concrete used in Palestine (B250-B500)

$$M_d = \frac{A_s d}{3}$$

Additional Requirements for Lower Limit on ρ

Temperature & Shrinkage reinforcement in structural slabs and footings (ACI 7.12) place perpendicular to direction of flexural reinforcement.

GR 40 or GR 50 Bars: A_s (T&S) = 0.0020 A_g

GR 60 A_s (T&S) = 0.0018 A_g

A_g - Gross area of the concrete

End of 4.2.1

Let Learning Continue

4.2.2 Beams: serviceability

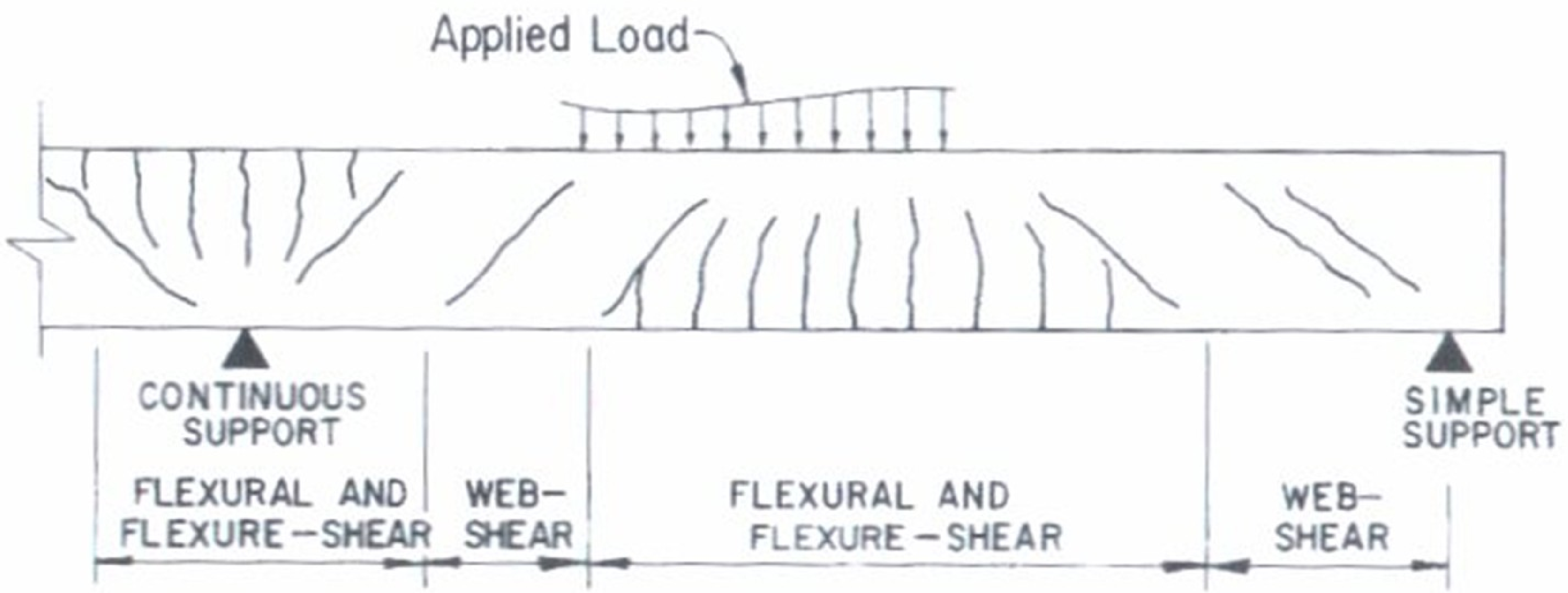
Beam Depths

• ACI 318 - Table 9.5(a)   min. h based on span (slab & beams)

•Design for max. moment over a support to set depth of a continuous beam.

4.2.3 Shear

Typical Crack Patterns for a deep beam.



Shear Design: review

Shear Strength (ACI 318 Sec 11.1)

$$\phi V_n \geq V_u$$

capacity ≥ demand

V_u = factored shear force at section

V_n = Nominal Shear Strength

ϕ = 0.75 (shear) – strength reduction factor

$$V_n = V_c + V_s$$

$V_c = \dfrac{\sqrt{f_c'}}{6} b_w d$ = Nominal shear resistance provided by concrete

V_s = Nominal shear resistance provided by the shear reinforcement

Shear Design: review\ Minimum Shear Reinforcement

11.4.6.1 — A minimum area of shear reinforcement, $A_{v,min}$, shall be provided in all reinforced concrete flexural members (prestressed and nonprestressed) where V_u exceeds $0.5\phi V_c$, except in members satisfying one or more of (a) through (f):

(a) Footings and solid slabs;

(c) Concrete joist construction defined by 8.13;

(d) Beams with h not greater than 250 mm;

(e) Beam integral with slabs with h not greater than 600 mm and not greater than the larger of 2.5 times thickness of flange, and 0.5 times width of web;

Approximate design for shear

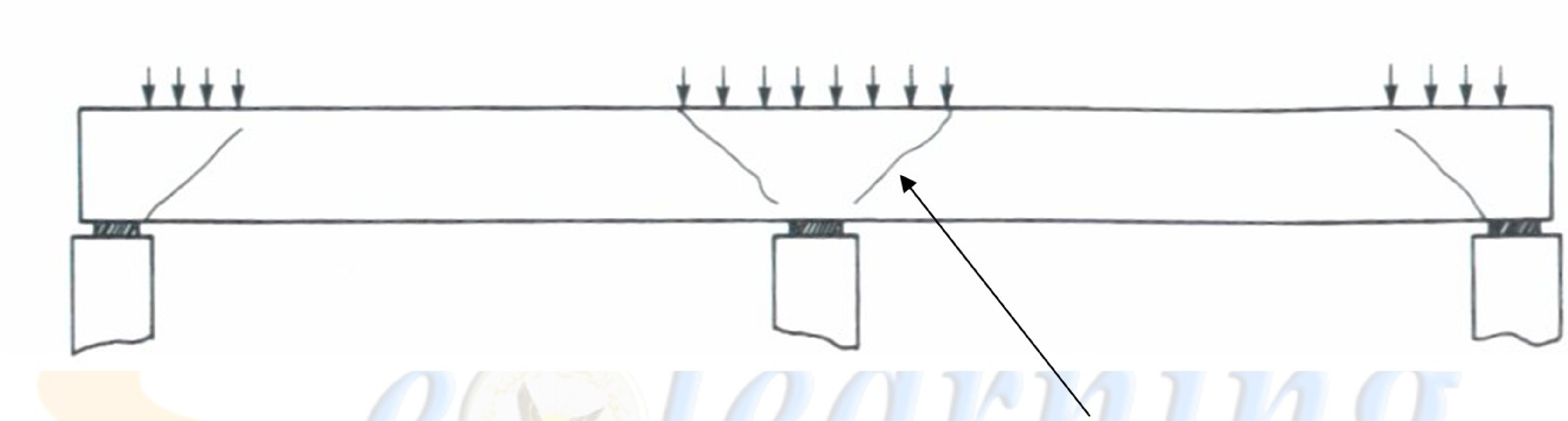
- Better to use

$$V_u < 0.5\phi\sqrt{f_c'}\ b_w d$$

- Hence

$$s_{\max} \le \frac{d}{2} \le 60cm$$

Shear Design: review/ max shear



Non-pre-stressed members:

Sections located less than a distance d from face of support may be designed for same shear, V_u, as the computed at a distance d.

Shear Design: review/ max shear

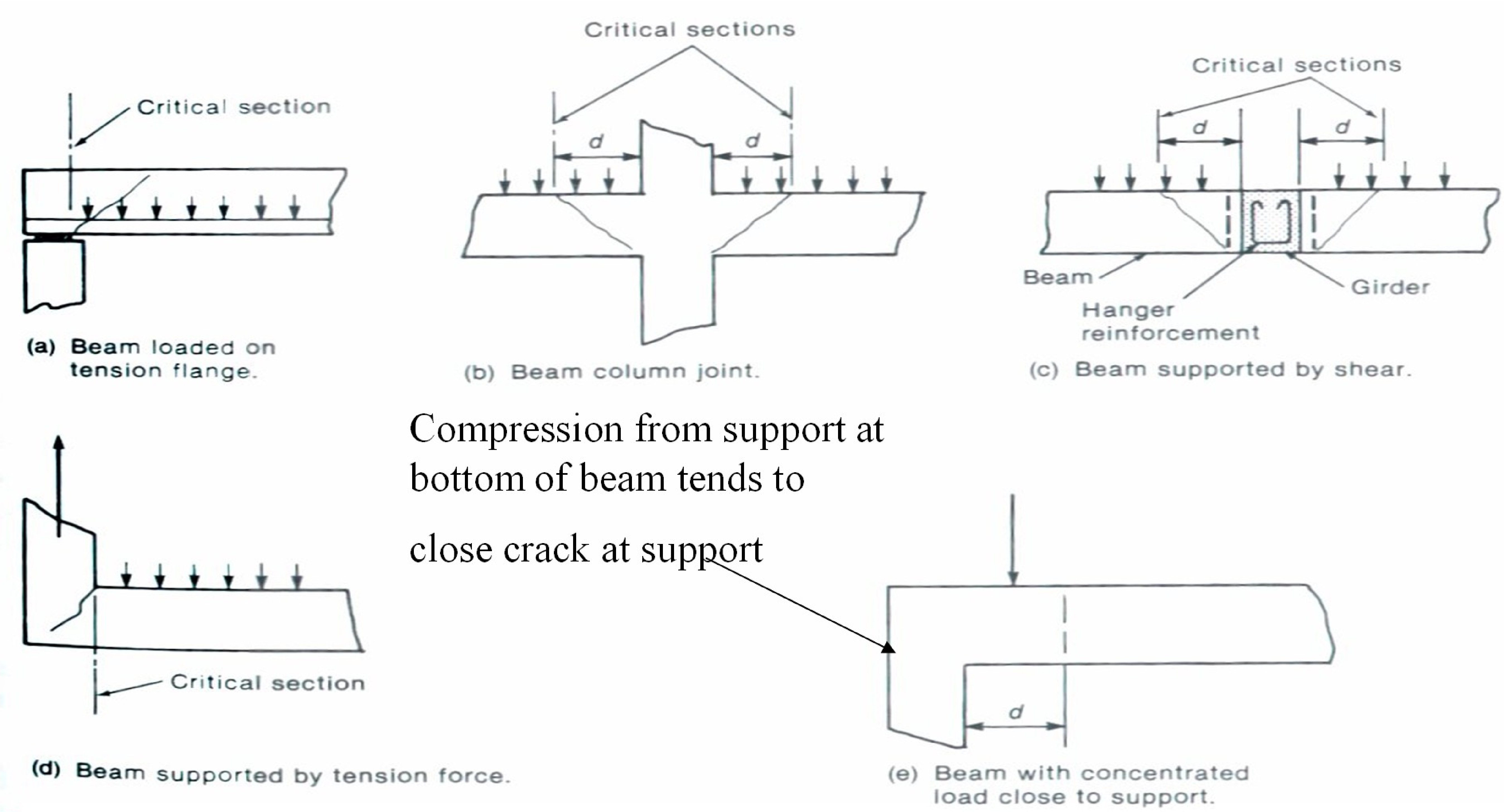
When:

1. The support reaction introduces compression into the end regions of the member.

2. The loads are applied at or near the top of the beam.

3. No concentrated load occurs with in d from face of support .

Shear Design: review/ max hear



Compression from support at bottom of beam tends to

close crack at support

4.2.4 Development Length

12.2.1 — Development length for deformed bars and deformed wire in tension, ℓ_d, shall be determined from either 12.2.2 or 12.2.3 and applicable modification factors of 12.2.4 and 12.2.5, but ℓ_d shall not be less than 300 mm.

12.2.2 — For deformed bars or deformed wire, ℓ_d shall be as follows:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\frac{f_y \psi_t \psi_e}{2.1 \lambda \sqrt{f'_c}}\right) d_b$	$\left(\frac{f_y \psi_t \psi_e}{1.7 \lambda \sqrt{f'_c}}\right) d_b$
Other cases	$\left(\frac{f_y \psi_t \psi_e}{1.4 \lambda \sqrt{f'_c}}\right) d_b$	$\left(\frac{f_y \psi_t \psi_e}{1.1 \lambda \sqrt{f'_c}}\right) d_b$

12.2.3 — For deformed bars or deformed wire, ℓ_d shall be

$$\ell_d = \left(\frac{f_y}{1.1 \lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right) d_b \qquad (12\text{-}1)$$

in which the confinement term $(c_b + K_{tr})/d_b$ shall not be taken greater than 2.5, and

$$K_{tr} = \frac{40 A_{tr}}{sn} \qquad (12\text{-}2)$$

where n is the number of bars or wires being spliced or developed along the plane of splitting. It shall be permitted to use $K_{tr} = 0$ as a design simplification

(a) Where horizontal reinforcement is placed such that more than 300 mm of fresh concrete is cast below the development length or splice, $\psi_t = 1.3$. For other situations, $\psi_t = 1.0$.

(c) For No. 19 and smaller bars and deformed wires, $\psi_s = 0.8$. For No. 22 and larger bars, $\psi_s = 1.0$.

4.2.5 Bar Splices in tension

Why do we need bar splices? -- for long spans

Types of Splices

1. Butted & Welded
2. Mechanical Connectors
3. Lab Splices

(Butted & Welded and Mechanical Connectors:) Must develop 125% of yield strength

Types of Splices

Class A Splice (ACI 12.15.2)

When $\dfrac{A_{s(\text{provided})}}{A_{s(\text{req'd})}} \geq 2$ over entire splice length.

and 1/2 or less of total reinforcement is spliced within the req’d lap length.

Types of Splices

Class B Splice (ACI 12.15.2)

All tension lab splices not meeting requirements of Class A Splices

Tension Lap Splice (ACI 12.15)

As,prov/As,req'd	%As Spliced	Splice Class	Lap, req'd	Notes
≥ 2.0	≤ 50	A	l_d	Desirable
	> 50	B	$1.3\ l_d$	ok
< 2.0	≤ 50	B	$1.3\ l_d$	ok
	> 50	B	$1.3\ l_d$	Avoid

where $A_{s\ (req'd)}$ = determined for bending

l_d = development length for bars (not allowed to use excess reinforcement modification factor)

l_d must be greater than or equal to 30cm

Lab Splices should be placed in away from regions of high tensile stresses -locate near points of inflection (ACI R12.15.2)

End of 4.2.2-5

Let Learning Continue