Chapter 2

Relativity

A Brief Overview of Modern Physics

- 20^{th} Century revolution
  - 1900 Max Planck
    - Basic ideas leading to Quantum theory
  - 1905 Einstein
    - Special Theory of Relativity
- 21^{st} Century
  - Story is still incomplete

Basic Problems

- Newtonian mechanics fails to describe properly the motion of objects whose speeds approach that of light
- Newtonian mechanics is a limited theory
  - It places no upper limit on speed
  - It is contrary to modern experimental results
  - Newtonian mechanics becomes a specialized case of Einstein’s special theory of relativity
    - When speeds are much less than the speed of light

Galilean Relativity

- To describe a physical event, a frame of reference must be established

- There is no absolute inertial frame of reference
  - This means that the results of an experiment performed in a vehicle moving with uniform velocity will be identical to the results of the same experiment performed in a stationary vehicle

Galilean Relativity, cont.

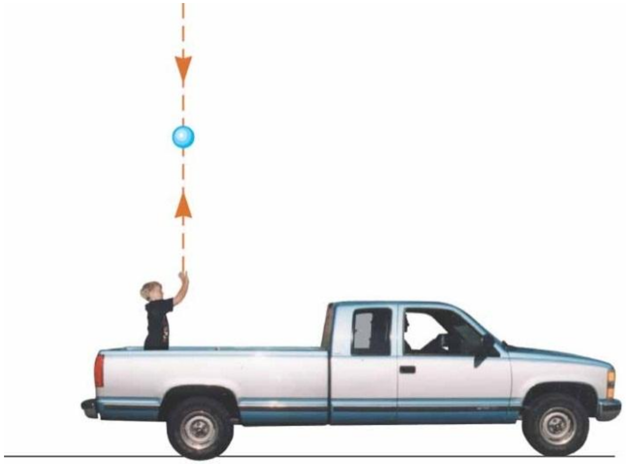
- Reminders about inertial frames
  - Objects subjected to **no forces** will experience

    No acceleration

  - Any system moving at **constant velocity** with respect to an inertial frame must also be in an **inertial frame**
- According to the **principle of Galilean relativity**, the laws of mechanics must be the **same** in all inertial frames of reference

Galilean Relativity – Example

- The observer in the truck throws a ball straight up
  - It appears to move in a vertical path
  - The law of gravity and equations of motion under uniform acceleration are obeyed



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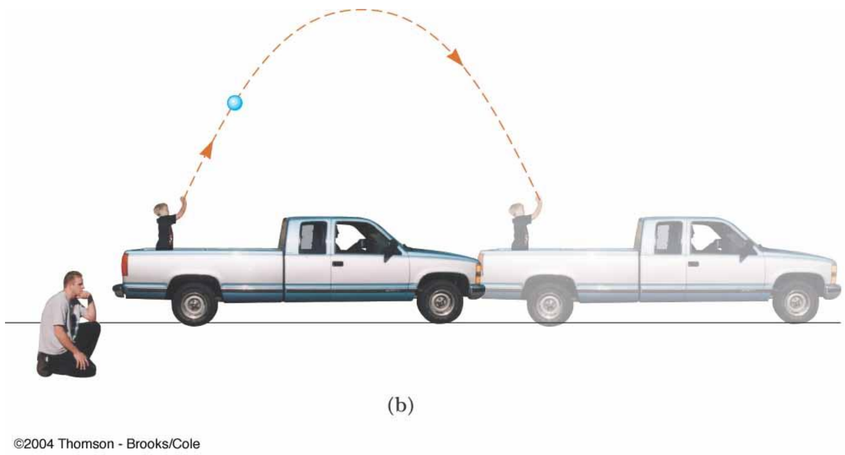
Galilean Relativity – Example, continue



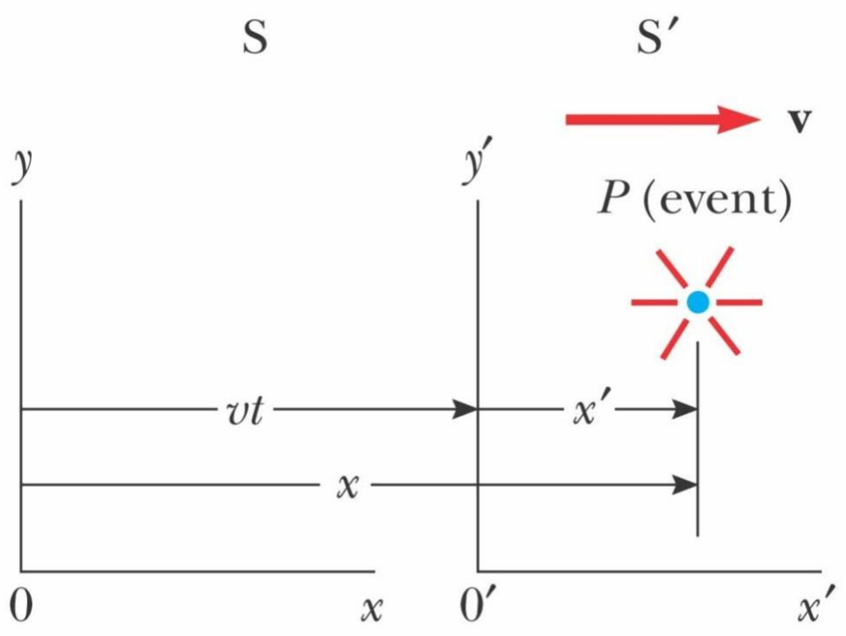
- There is a stationary observer on the ground
  - Views the path of the ball thrown to be a parabola
  - The ball has a velocity to the right **equal** to the velocity of the truck

Galilean Relativity – Example, conclusion

- The two observers disagree on the shape of the ball's path
- Both agree that the motion obeys the law of gravity and Newton's laws of motion
- Both agree on how long the ball was in the air
- Conclusion: There is no preferred frame of reference for describing the laws of mechanics

Views of an Event

- An *event* is some physical phenomenon
- Assume the event occurs and is observed by an observer at rest in an inertial reference frame
- The event's location and time can be specified by the coordinates (x, y, z, t)



Views of an Event, cont.

- Consider two inertial frames, S and S´
- S' moves with constant velocity, $\vec{\mathbf{v}}$, along the common x and x´ axes
- The velocity is measured relative to S
- Assume the origins of S and S´ coincide at $t = 0$

Galilean Space-Time Transformation Equations

- An observer in S describes the event with space-time coordinates (x, y, z, t)
- An observer in S' describes the same event with space-time coordinates (x´, y´, z´, t´)
- The relationship among the coordinates are
  - $x' = x - vt$
  - $y' = y$
  - $z' = z$
  - $t' = t$

Notes About Galilean Transformation Equations

- The time is the **same** in both inertial frames
  - Within the framework of classical mechanics, all clocks run at the same rate
  - The time at which an event occurs for an observer in S is the same as the time for the same event in S´
  - **This turns out to be incorrect when v is comparable to the speed of light**

Galilean Velocity Transformation Equation

- Suppose that a particle moves through a displacement *dx* along the *x* axis in

  a time *dt*
- The corresponding displacement dx´ is

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

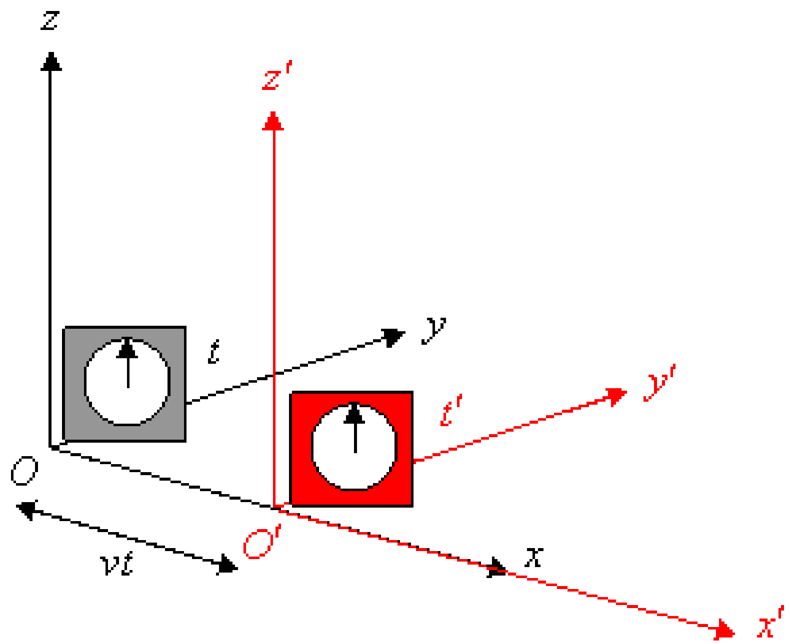
$$\text{or} \quad u'_x = u_x - v$$

- $\vec{\mathbf{u}}$ is used for the particle velocity and $\vec{\mathbf{v}}$ is used for the relative velocity between the two frames

Galilean Transformation Equations – Final Notes

- The x and x´ axes coincide, but their origins are different
- The y and y´ axes are parallel, but do not coincide
  - This is due to the displacement of the origin of S´ with respect to that of S
  - The same holds for z and z´ axes

- Time = 0 when the origins of the two coordinate system coincide
- If the S´ frame is moving in the positive x direction relative to S, the v is **positive**
  - Otherwise, it is **negative**



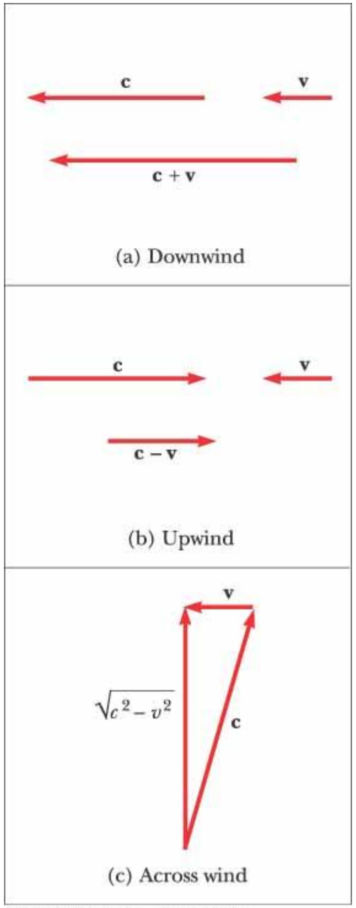
Two frames of reference relatively displaced along the *x*-axis

Speed of Light

- Galilean relativity does not apply to electricity, magnetism, or optics
- Maxwell showed the speed of light in free space is $c = 3.00 \times 10^8$ m/s
- Physicists in the late 1800s thought light moved through a medium called the ***ether***
  - The speed of light would be ***c*** only in a special, absolute frame at rest with respect to the ether

Effect of Ether Wind on Light

- Assume *v* is the velocity of the ether wind relative to the earth
- *c* is the speed of light relative to the ether
- Various resultant velocities are shown



Ether Wind, cont.

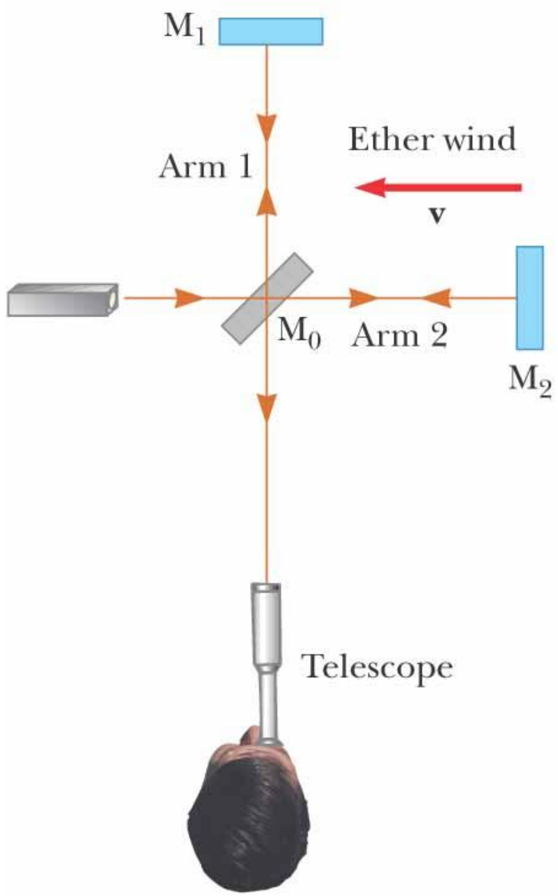
- The velocity of the ether wind is assumed to be the orbital velocity of the Earth
- All attempts to detect and establish the existence of the ether wind proved futile
- But Maxwell’s equations seem to imply that the speed of light always has a fixed value in all inertial frames
  - This is a contradiction to what is expected based on the Galilean velocity transformation equation

Michelson-Morley Experiment

- First performed in 1881 by Michelson
- Repeated under various conditions by Michelson and Morley
- Designed to detect small changes in the speed of light
  - By determining the velocity of the Earth relative to the ether

Michelson-Morley Equipment

- Used the Michelson interferometer
- Arm 2 is aligned along the direction of the Earth's motion through space
- The interference pattern was observed while the interferometer was rotated through 90°
- The effect should have been to show small, but measurable shifts in the fringe pattern



Michelson-Morley Expected Results

- The speed of light measured in the Earth frame should be $c - v$ as the light approaches mirror M_2
- The speed of light measured in the Earth frame should be $c + v$ as the light is reflected from mirror M_2
- The experiment was repeated at different times of the year when the ether wind was expected to change direction and magnitude

Michelson-Morley Results

- Measurements failed to show any change in the fringe pattern
  - No fringe shift of the magnitude required was ever observed
  - The negative results contradicted the ether hypothesis
  - They also showed that it was impossible to measure the absolute velocity of the Earth with respect to the ether frame
- Light is now understood to be an electromagnetic wave, which requires **No** medium for its propagation
  - **The idea of an ether was discarded**

Albert Einstein

- 1879 – 1955
- 1905
  - Special theory of relativity
- 1916
  - General relativity
  - 1919 – confirmation
- 1920’s
  - Didn’t accept quantum theory
- 1940’s or so
  - Search for unified theory - unsuccessful

Einstein's Principle of Relativity

- Resolves the contradiction between Galilean relativity and the fact that the **speed of light** is the **same** for all observers
- Postulates
  - **The principle of relativity**: The laws of physics must be the same in all inertial reference frames
  - **The constancy of the speed of light**: the speed of light in a vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light

The Principle of Relativity

- This is a generalization of the principle of Galilean relativity, which refers only to the laws of mechanics
- The results of any kind of experiment performed in a laboratory at **rest** must be
- the same as when performed in a laboratory moving at a **constant speed** past the first one
- No preferred inertial reference frame exists
- It is impossible to detect absolute motion

The Constancy of the Speed of Light

- This is required by the first postulate
- Confirmed experimentally in many ways
- Explains the null result of the Michelson-Morley experiment
- **Relative** motion is unimportant when measuring the speed of light
  - We must alter our common-sense notions of space and time

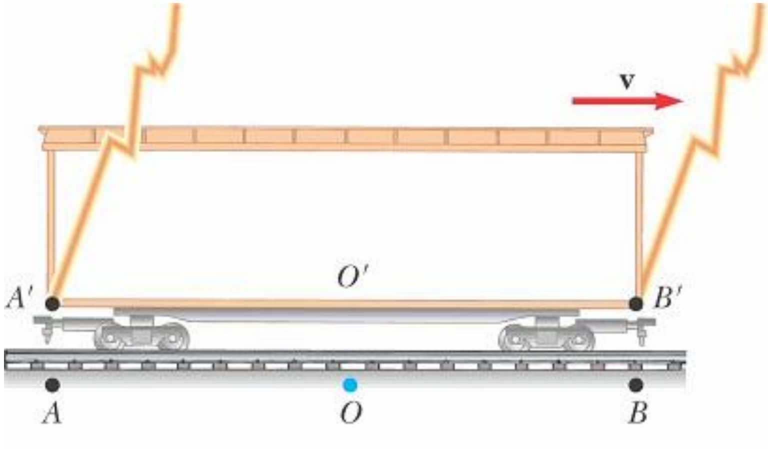
Consequences of Special Relativity

- Restricting the discussion to concepts of simultaneity, time intervals, and length
  - These are quite different in relativistic mechanics from what they are in Newtonian mechanics
- In relativistic mechanics
  - **There is no such thing as absolute length**
  - **There is no such thing as absolute time**
  - Events at different locations that are observed to occur simultaneously in one frame are **Not** observed to be simultaneous in another frame moving uniformly past the first

Simultaneity((happening at the same time)

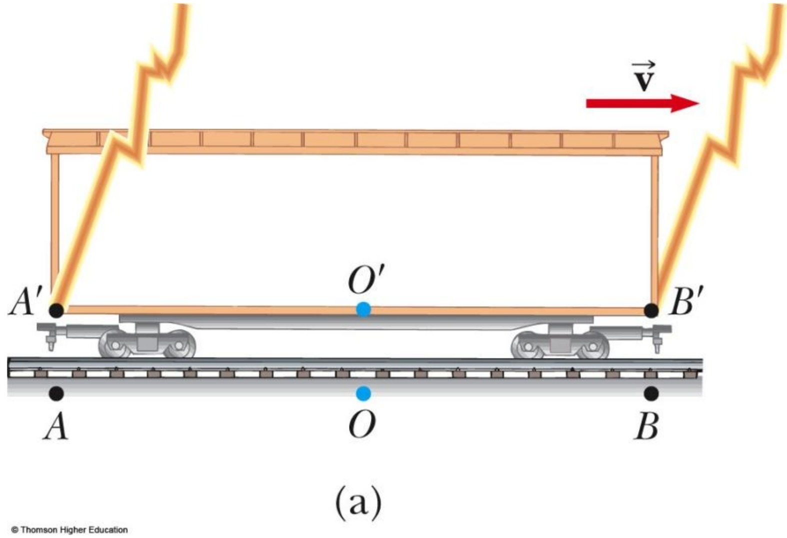
- In special relativity, Einstein abandoned the assumption of simultaneity
- Thought experiment to show this
  - A boxcar moves with uniform velocity
  - Two lightning bolts strike the ends
  - The lightning bolts leave marks (A´ and B´) on the car and (A and B) on the ground
  - Two observers are present: O´ in the boxcar and O on the ground

Simultaneity – Thought Experiment Set-up



- Observer O is midway between the points of lightning strikes on the ground, A and B
- Observer O´ is midway between the points of lightning strikes on the boxcar, A´ and B´

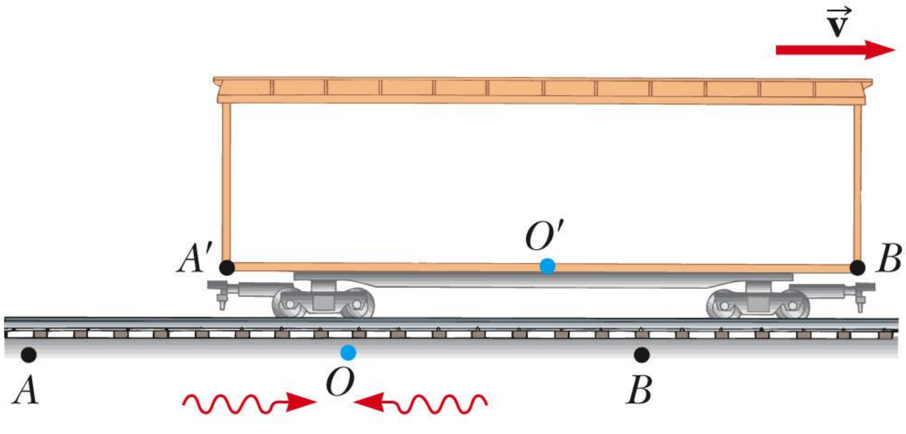
Simultaneity – Thought Experiment Results



- The light reaches observer O at the same time
  - He concludes the light has traveled at the **same speed over equal distances**
  - Observer O concludes the lightning bolts occurred **simultaneously**

Simultaneity – Thought Experiment Results, cont.

- By the time the light has reached observer *O*, observer
- *O´* has moved
- The signal from *B´* has already swept past *O´*, but the signal from *A´* has not yet reached him
  - The two observers must find that light travels at the **same speed**
  - Observer *O´* concludes the **lightning struck the front of the boxcar before it struck the back (they were not simultaneous events**)



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Simultaneity – Thought Experiment, Summary

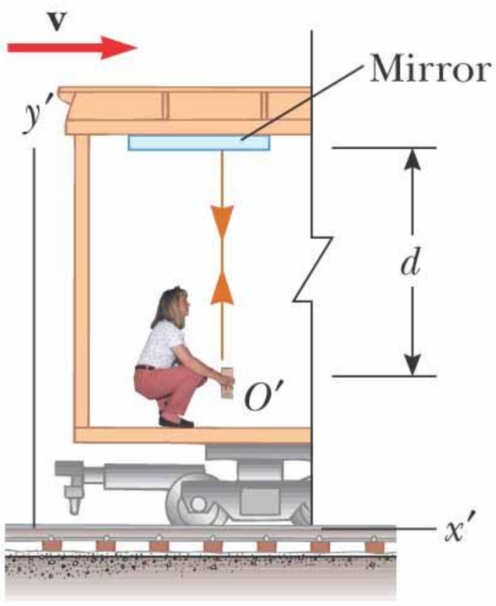
- Two events that are simultaneous in one reference frame are in general **Not** simultaneous in a second reference frame moving relative to the first
- That is, simultaneity is **Not** an absolute concept, but rather one that depends on the **state of motion of the observer**
  - In the thought experiment, both observers are correct, because there is **No** preferred inertial reference frame

Simultaneity, Transit Time

- In this thought experiment, the **disagreement** depended upon the **transit time of light** to the observers and doesn't demonstrate the deeper meaning of relativity
- In **high-speed** situations, the **simultaneity** is **relative** even when transit time is subtracted out
  - We will ignore transit time in all further discussions

Time Dilation

- A mirror is fixed to the ceiling of a vehicle
- The vehicle is moving to the right with speed ***v***
- An observer, *O´*, at rest in the frame attached to the vehicle holds a flashlight a distance *d* below the mirror
- The flashlight emits a pulse of light directed at the mirror (event 1) and the pulse arrives back after being reflected (event 2)



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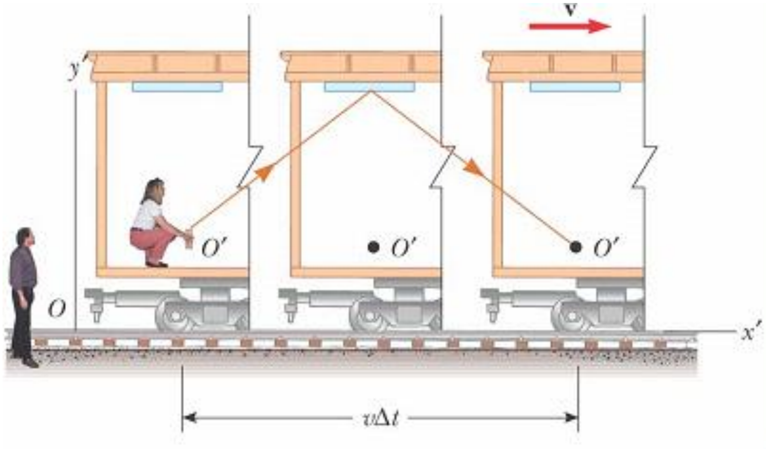
Time Dilation, Moving Observer

- Observer O' carries a clock
- She uses it to measure the time between the events (Δt_p)
  - She observes the events to occur at the same place
  - **Δt_p = distance/speed = $(2d)/c$**

Time Dilation, Stationary Observer



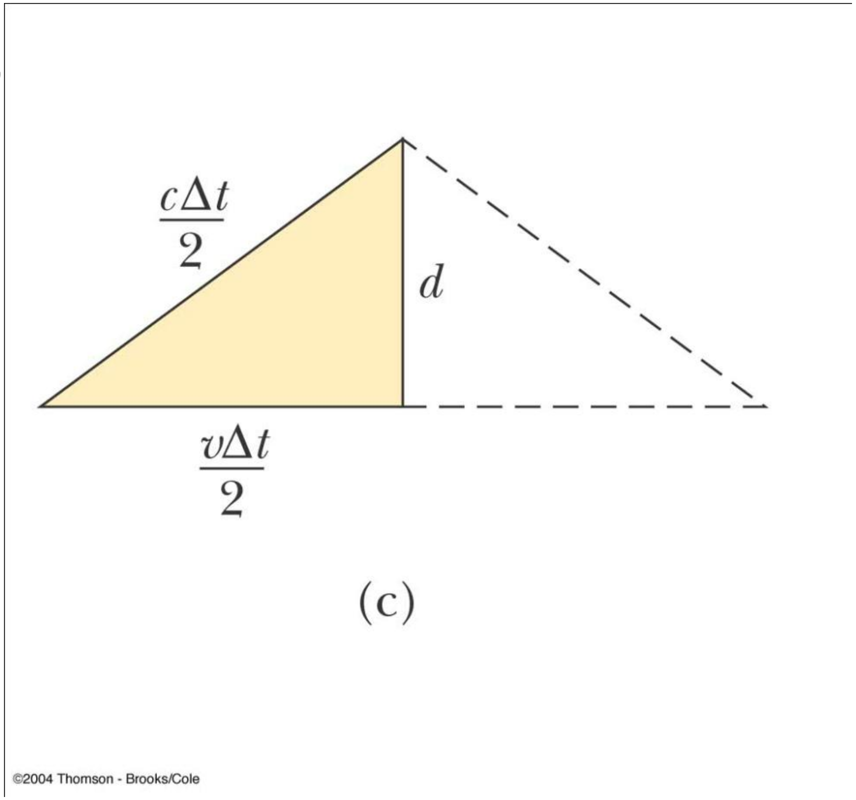
- Observer O is a stationary observer on the Earth
- He observes the mirror and O´ to move with speed *v*
- By the time the light from the flashlight reaches the mirror, the mirror has moved to the right
- The light must **travel farther with respect to O than** with respect to O´

Time Dilation, Observations

- Both observers must measure the speed of the light to be ***c***
- The light travels farther for O
- The time interval, $\boldsymbol{\Delta t}$**, for O** is **longer** than the time interval for **O´**, $\boldsymbol{\Delta t_p}$

Time Dilation, Time Comparisons

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_p$$



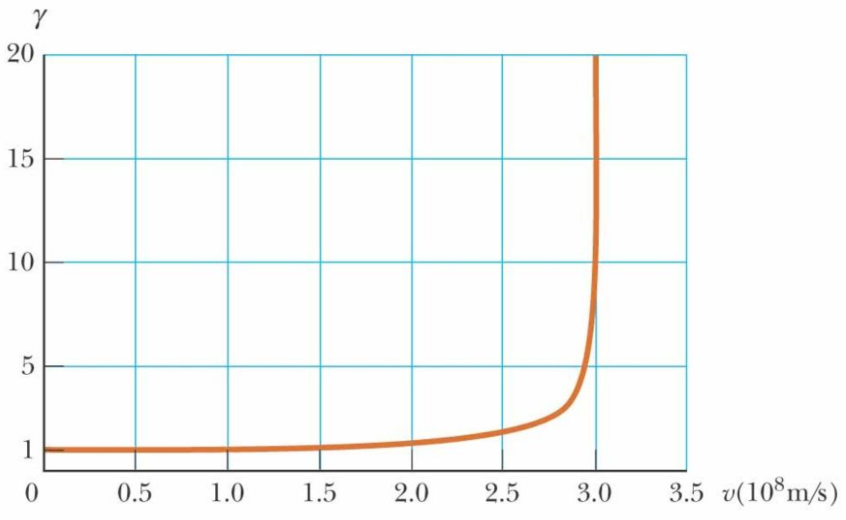
$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time Dilation, Summary

- The time interval Δt between two events measured by an observer **moving** with respect to a clock is **longer** than the time interval Δt_p between the same two events measured by an observer at **rest** with respect to the clock
  - This effect is known as **time dilation**

γ Factor

- Time dilation is not observed in our everyday lives
- For **slow** speeds, the factor of γ is so small that **No** time dilation occurs
- As the speed approaches the speed of light, γ increases rapidly



Identifying Proper Time

- The time interval Δt_p is called the **proper time** interval
  - The proper time interval is the time interval between events as measured by an observer who sees the events occur at the **same point** in space
    - You must be able to correctly identify the observer who measures the proper time interval

Time Dilation – Generalization

- If a clock is **moving** with respect to you, the time interval between ticks of the moving clock is observed to be **longer** than the time interval between ticks of an identical clock in your reference frame
- All physical processes are measured to slow down when these processes occur in a frame moving with respect to the observer
  - These processes can be chemical and biological as well as physical

Time Dilation – Verification

- Time dilation is a very real phenomenon that has been verified by various experiments
- These experiments include:
  - Airplane flights
  - Muon decay
  - Twin Paradox

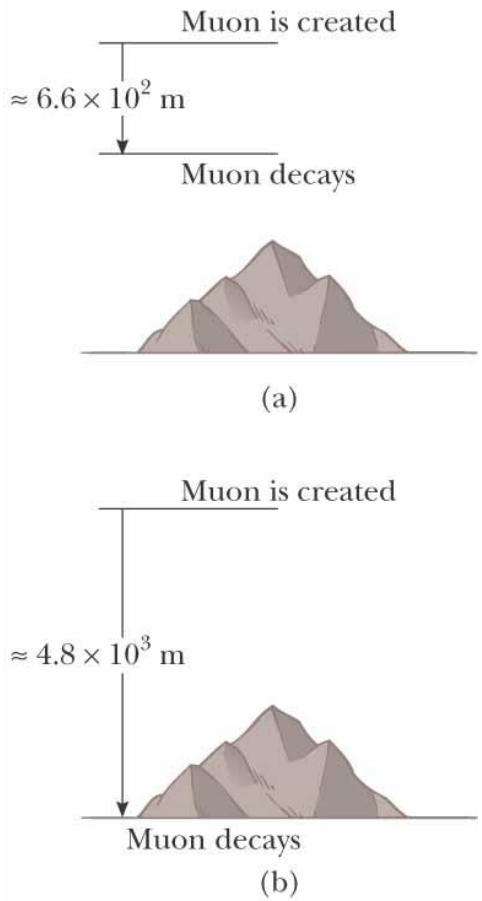
Airplanes and Time Dilation

- In 1972 an experiment was reported that provided direct evidence of time dilation
- Time intervals measured with four cesium clocks in jet flight were compared to time intervals measured by Earth-based reference clocks
- The results were in good agreement with the predictions of the special theory of relativity

Time Dilation Verification – Muon Decays

- Muons are unstable particles that have the same charge as an electron, but a mass 207 times more than an electron
- Muons have a half-life of $\Delta t_p = 2.2\ \mu s$ when measured in a reference frame at rest with respect to them (a)
- Relative to an observer on the Earth, muons should have a lifetime of

  $\gamma\, \Delta t_p$ (b)
- A CERN experiment measured lifetimes in agreement with the predictions of relativity



The Twin Paradox – The Situation

- A thought experiment involving a set of twins, Speedo and Goslo
- Speedo travels to Planet X,

  Which is 20 light years from the Earth
  - His ship travels at 0.95c
  - After reaching Planet X, he immediately returns to the Earth at the same speed
- When Speedo returns, he has aged 13 years, but Goslo has aged 42 years

The Twins' Perspectives

- Goslo's perspective is that he was at **rest** while Speedo went on the journey
- Speedo thinks he was at rest and Goslo and the Earth **raced away** from him and then headed back toward him
- The paradox – which twin has developed signs of excess aging ?

The Twin Paradox – The Resolution

- Relativity applies to reference frames moving at uniform speeds
- The trip in this thought experiment is not symmetrical since Speedo must experience a series of accelerations during the journey
- Therefore, Goslo can apply the time dilation formula with a proper time of 42 years
  - This gives a time for Speedo of 13 years and this agrees with the earlier result
- There is no true paradox since Speedo is not in an inertial frame

Length Contraction

- The measured distance between two points depends on the frame of reference of the observer
- The proper length, $\mathbf{L_p}$, of an object is the length of the object measured by someone at **rest** relative to the object
- The length of an object measured in a reference frame that is **moving** with respect to the object is always **less** than the proper length
  - This effect is known as length contraction

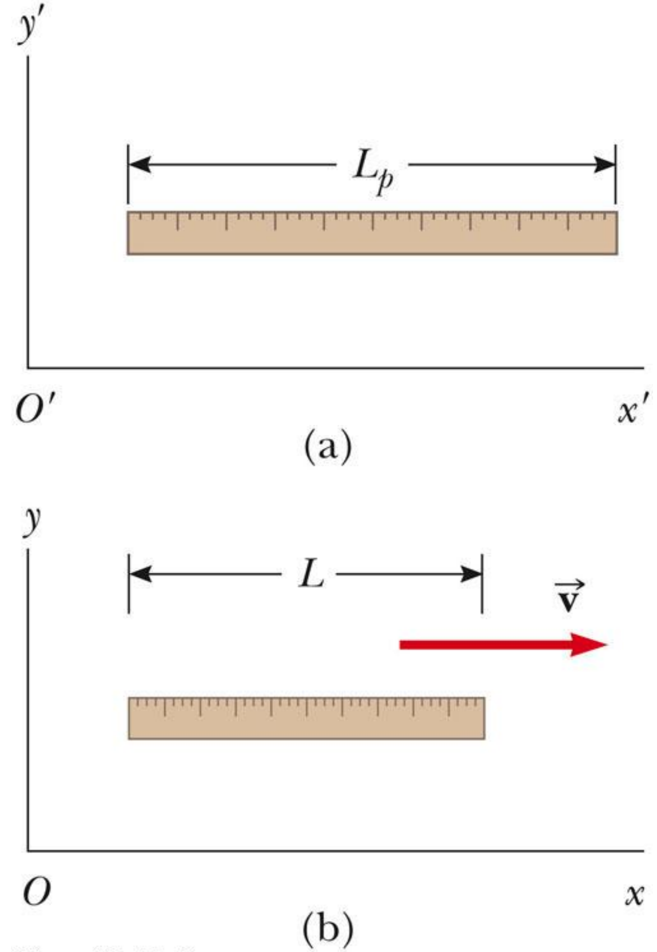
More About Proper Length

- Very important to correctly identify the observer who measures proper length
- The proper length is always the length measured by the observe at rest with respect to the points
- Often the proper time interval and the proper length are ***not*** measured by the same observer

Length Contraction – Equation

$$L = \frac{L_P}{\gamma} = L_P\sqrt{1 - \frac{v^2}{c^2}}$$

- Length contraction takes place only along the direction of motion

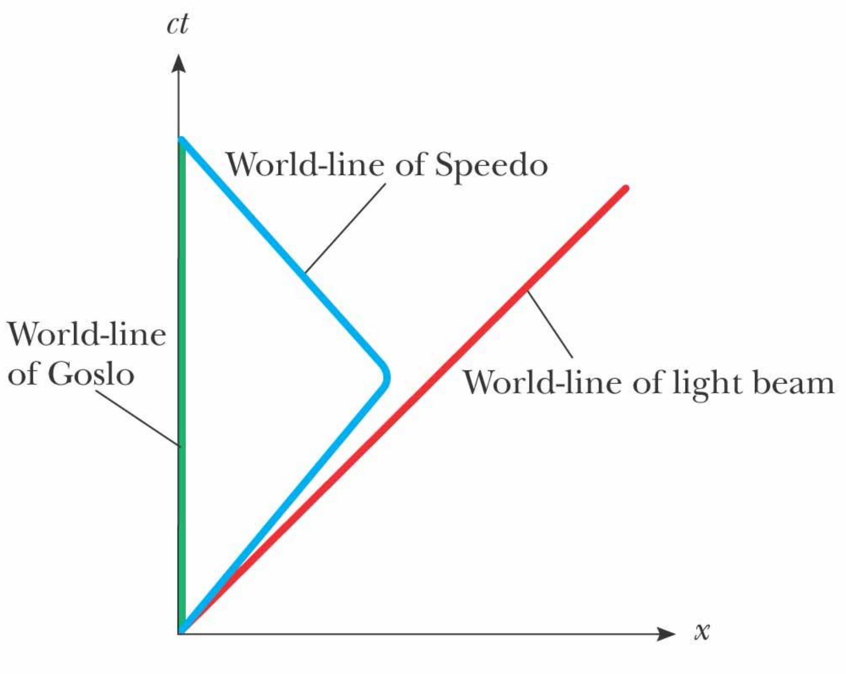


Proper Length vs. Proper Time

- The proper length and proper time interval are defined differently
- The proper length is measured by an observer for whom the **end points** of the length remained **fixed** in space
- The proper time interval is measured by someone for whom the two events take place at the **same position** in space

Space-Time Graphs

- In a *space-time graph*, *ct* is the ordinate and position *x* is the abscissa
- The example is the graph of the twin paradox
- A path through space-time is called a **world-line**
- World-lines for light are diagonal lines



Relativistic Doppler Effect

- Another consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest
- If a light source and an observer approach each other with a relative speed, *v*, the frequency measured by the observer is

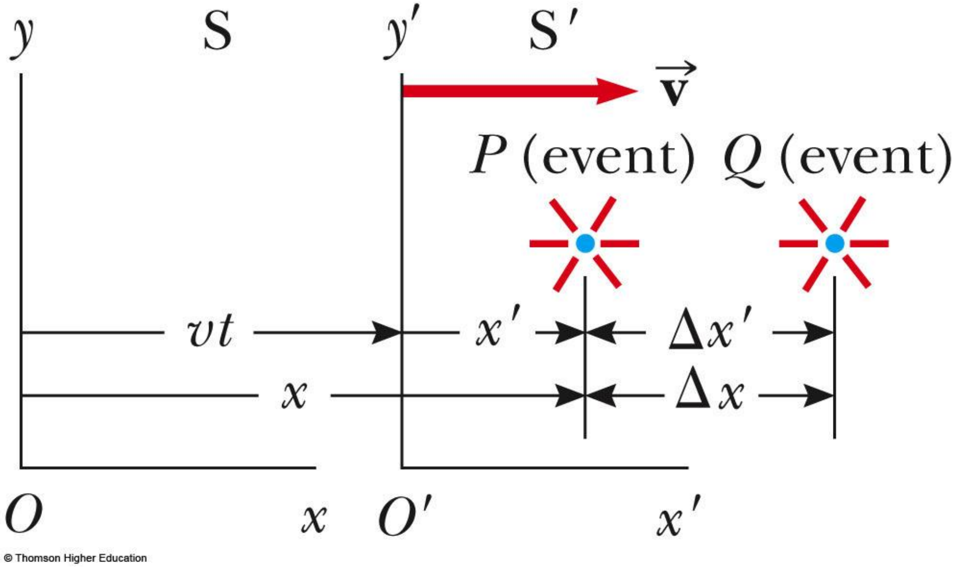
$$f_{\text{obs}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{\text{source}}$$

Relativistic Doppler Effect, cont.

- The frequency of the **source** is measured in its **rest** frame
- The shift depends only on the relative velocity, v, of the source and observer
- $f_{obs} > f_{source}$ when the source and the observer **approach** each other
- An example is the red shift of galaxies, showing most galaxies are moving away from us

Lorentz Transformation Equations, Set-Up

- Assume the events at points *P* and *Q* are reported by two observers
- One observer is at rest in frame S
- The other observer is in frame S' moving to the right with speed *v*



Lorentz Transformation Equations, Set-Up, cont.

- The observer in frame S reports the event with space-time coordinates of (x, y, z, t)
- The observer in frame S' reports the same event with space-time coordinates of (x', y', z', t')
- The Galilean transformation would predict that $\Delta x = \Delta x'$
  - The distance between the two points in space at which the events occur does not depend on the motion of the observer

Lorentz Transformations Compared to Galilean

- The Galilean transformation is not valid when *v* approaches *c*
  - $\Delta x = \Delta x'$ is contradictory to length contraction
- The equations that are valid at all speeds are the Lorentz transformation equations
  - Valid for speeds $0 < v < c$

Lorentz Transformations, Equations

- To transform coordinates from S to S' use

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

$$\text{or} \quad u'_x = u_x - v$$

- These show that in relativity, space and time are not separate concepts but rather closely interwoven with each other
- To transform coordinates from S' to S use

$$x = \gamma(x' + vt') \quad y' = y \quad z' = z \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Lorentz Transformations, Pairs of Events

- The Lorentz transformations can be written in a form suitable for describing pairs of events
- For S to S' For S' to S

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$

$$\Delta t = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)$$

Lorentz Transformations, Pairs of Events, cont.

- In the preceding equations, observer O' measures $\Delta x' = x'_2 - x'_1$ and $\Delta t' = t'_2 - t'_1$
- Also, observer O measures $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$
- The y and z coordinates are unaffected by the motion along the x direction

Lorentz Velocity Transformation

- The “event” is the motion of the object
- S’ is the frame moving at v relative to S
- In the S’ frame

$$u_x' = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \quad \text{and} \quad u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_z v}{c^2}\right)}$$

Lorentz Velocity Transformation, cont.

- The term v does not appear in the u'_y and u'_z equations since the relative motion is in the x direction
- When v is much smaller than c, the Lorentz velocity transformation reduces to the Galilean velocity transformation equation
- If v = c, u'x = c and the speed of light is shown to be independent of the relative motion of the frame

Lorentz Velocity Transformation, final

- To obtain u_x in terms of u'_x, use

$$u_x = \frac{u'_x + v}{1 + \dfrac{u'_x v}{c^2}}$$

Measurements Observers Do Not Agree On

- Two observers *O* and *O'* do not agree on:
  - The time interval between events that take place in the same position in one reference frame
  - The distance between two points that remain fixed in one of their frames
  - The velocity components of a moving particle
  - Whether two events occurring at different locations in both frames are simultaneous or not

Measurements Observers Do Agree On

- Two observers *O* and *O'* can agree on:
  - Their relative speed of motion *v* with respect to each other
  - The speed c of any ray of light
  - The simultaneity of two events which take place at the same position and time in some frame

Relativistic Linear Momentum

- To account for conservation of momentum in all inertial frames, the definition must be modified to satisfy these conditions
  - The linear momentum of an isolated particle must be conserved in all collisions
  - The relativistic value calculated for the linear momentum p of a particle must approach the classical value m*u* as *u* approaches zero

$$\vec{\mathbf{p}} \equiv \frac{m\vec{\mathbf{u}}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m\vec{\mathbf{u}}$$

- $\vec{u}$ is the velocity of the particle, m is its mass

Mass in Relativity

- In older treatments of relativity, conservation of momentum was maintained by using "relativistic mass"
- Today, mass is considered to be ***invariant*** (same). That means it is i**ndependent** of speed
- The mass of an object in **all** frames is considered to be the **mass** as measured by an observer at **rest** with respect to the object

Relativistic Form of Newton's Laws

- The relativistic force acting on a particle whose linear momentum is $\vec{\mathbf{p}}$ is defined as

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

- This preserves classical mechanics in the limit of low velocities
- It is consistent with conservation of linear momentum for an isolated system both relativistically and classically
- Looking at acceleration it is seen to be impossible to accelerate a particle from rest to a **speed u ≥ c**

Speed of Light, Notes

- The speed of light is the speed limit of the universe
- It is the maximum speed possible for energy and information transfer
- Any object with **mass** must move at **a lower** speed

Relativistic Energy

- The definition of kinetic energy (K) requires modification in relativistic mechanics
- $K = \gamma mc^2 - mc^2$
  - This matches the classical kinetic energy equation when u << c
  - The term $\mathbf{mc^2}$ is called the rest energy of the object and is **independent of its speed**
  - The term $\boldsymbol{\gamma}\mathbf{mc^2}$ **is the total energy**, E, of the object and depends on its **speed** and its
  - **rest energy**

Relativistic Kinetic Energy

- The Work-Kinetic Energy Theorem can be applied to relativistic situations
- This becomes

$$W = K = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2 =$$

$$\gamma mc^2 - mc^2 = (\gamma - 1) mc^2$$

Relativistic Energy – Consequences

- A particle has energy by virtue (good quality) of its mass alone
  - A stationary particle with zero kinetic energy has an energy proportional to its inertial mass
  - This is shown by $E = K + mc^2 = 0 + mc^2$
- A small mass corresponds to an enormous amount of energy

Mass and Energy

- This is also used to express masses in energy units
  - Mass of an electron = 0.511 MeV/c^2
  - Conversion: 1 u = 931.50 MeV/c^2
- When using Conservation of Energy, **rest energy** must be included as another form of energy storage
- The conversion from mass to energy is useful in **nuclear reactions**