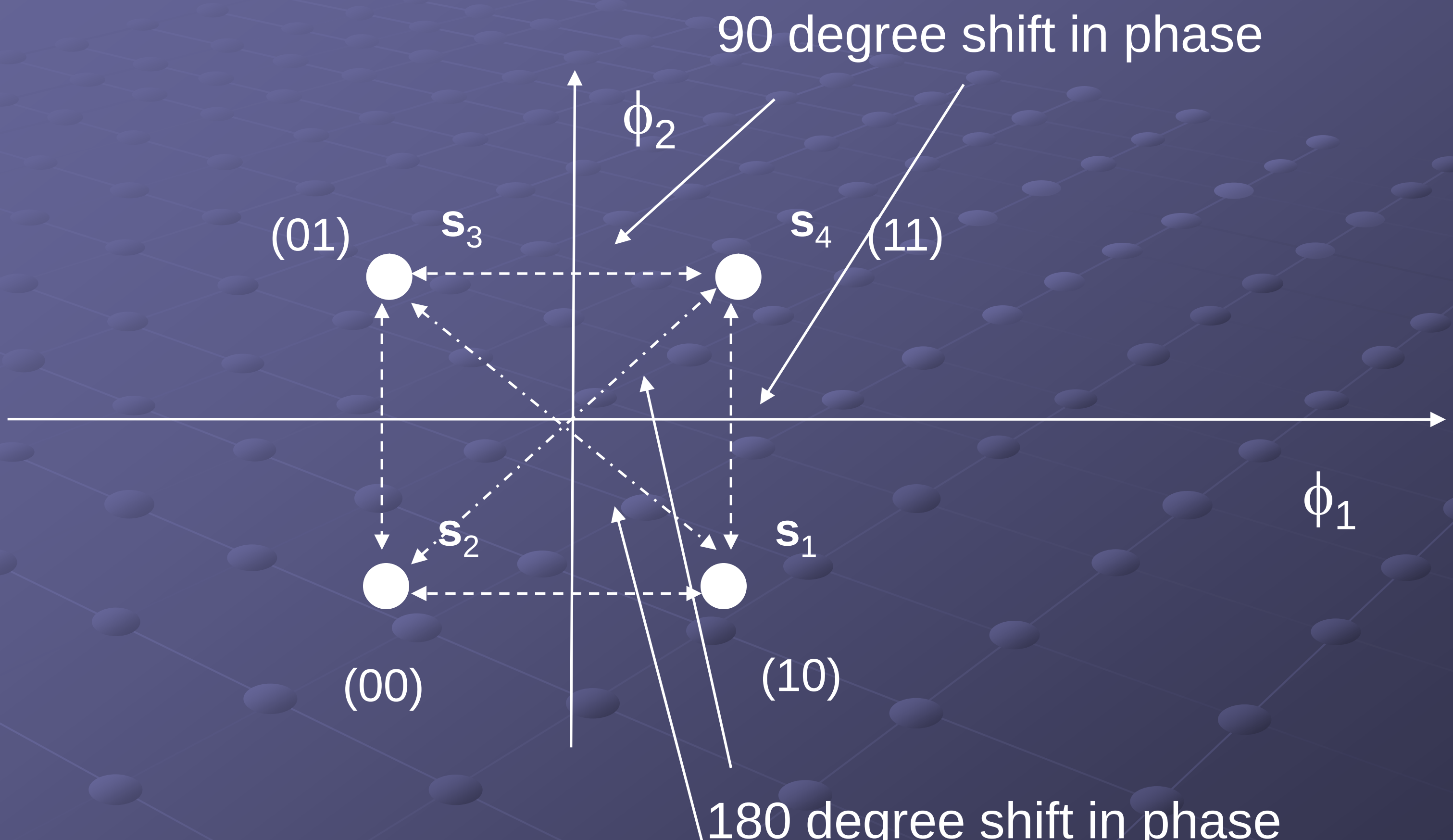
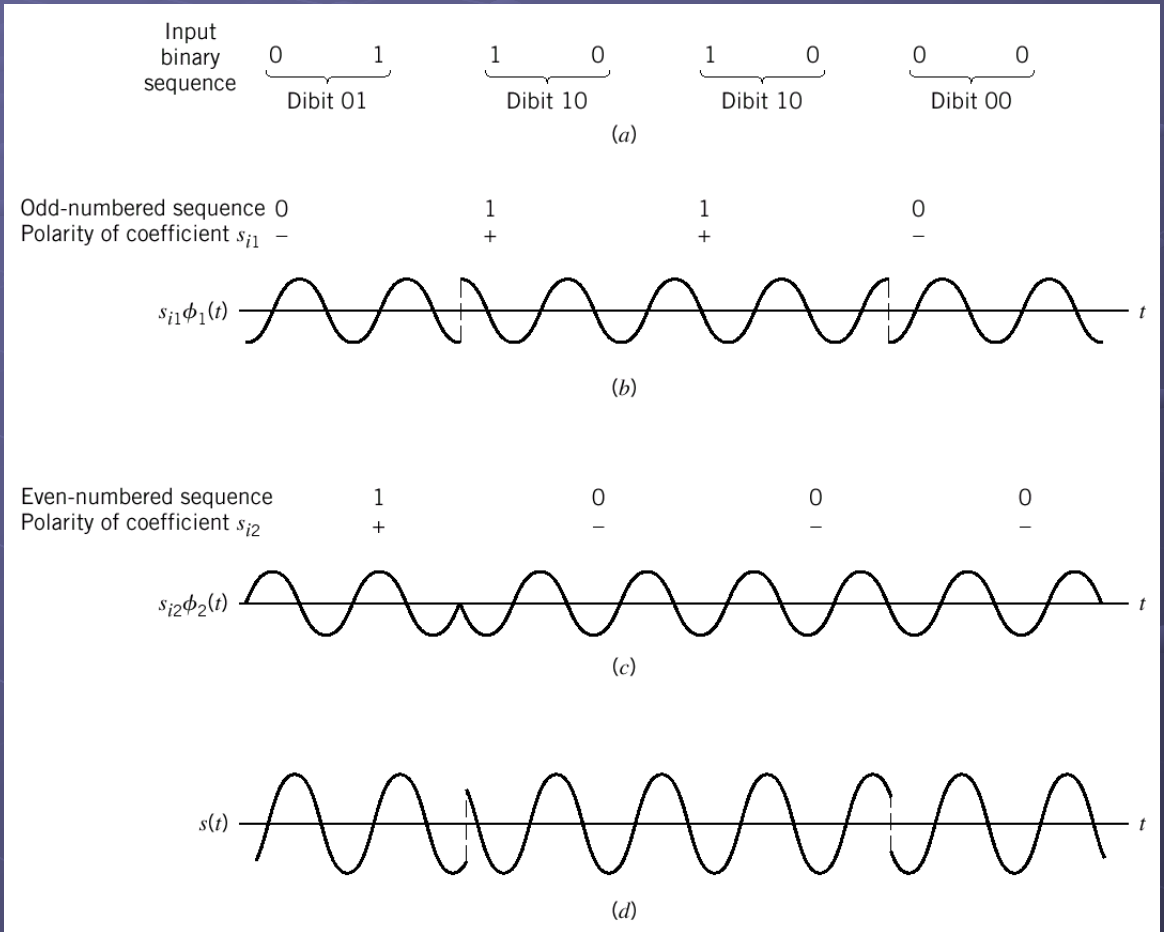
OFFSET QPSK


90 degree shift in phase
ϕ_2
s_3
s_4
(01)
(11)
ϕ_1
s_2
s_1
(00)
(10)
180 degree shift in phase

OFFSET QPSK

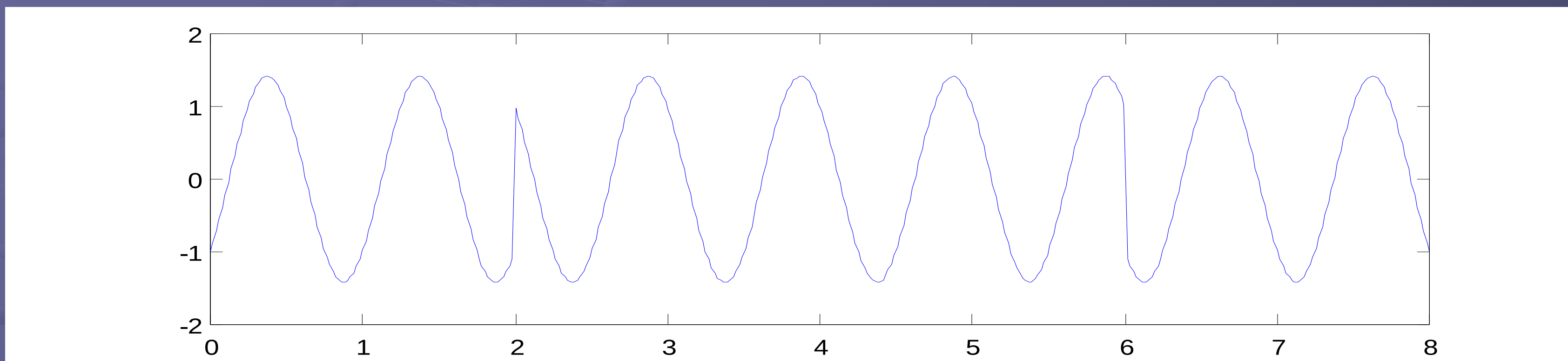


Input binary sequence
0 1
Dibit 01
1 0
Dibit 10
1 0
Dibit 10
0 0
Dibit 00
(a)
Odd-numbered sequence 0 1 1 0
Polarity of coefficient s_{i1} – + + –
$s_{i1}\phi_1(t)$
t
(b)
Even-numbered sequence 1 0 0 0
Polarity of coefficient s_{i2} + – – –
$s_{i2}\phi_2(t)$
t
(c)
$s(t)$
t
(d)

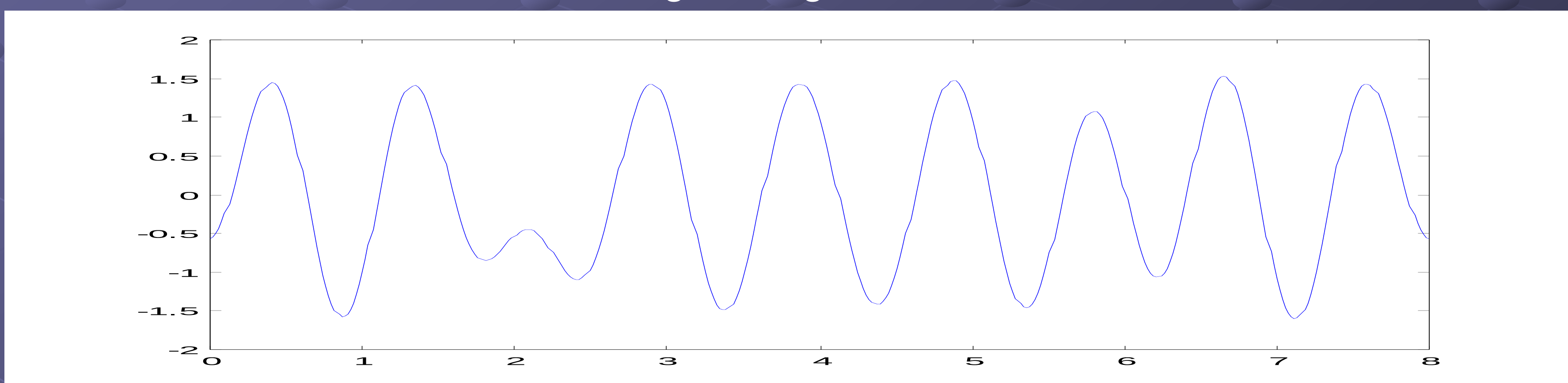
OFFSET QPSK

- Whenever both bits are changed simultaneously, 180 degree phase-shift occurs.
- At 180 phase-shift, the amplitude of the transmitted signal changes very rapidly costing amplitude fluctuation.
- This signal may be distorted when is passed through the filter or nonlinear amplifier.

OFFSET QPSK



Original Signal



Filtered signal

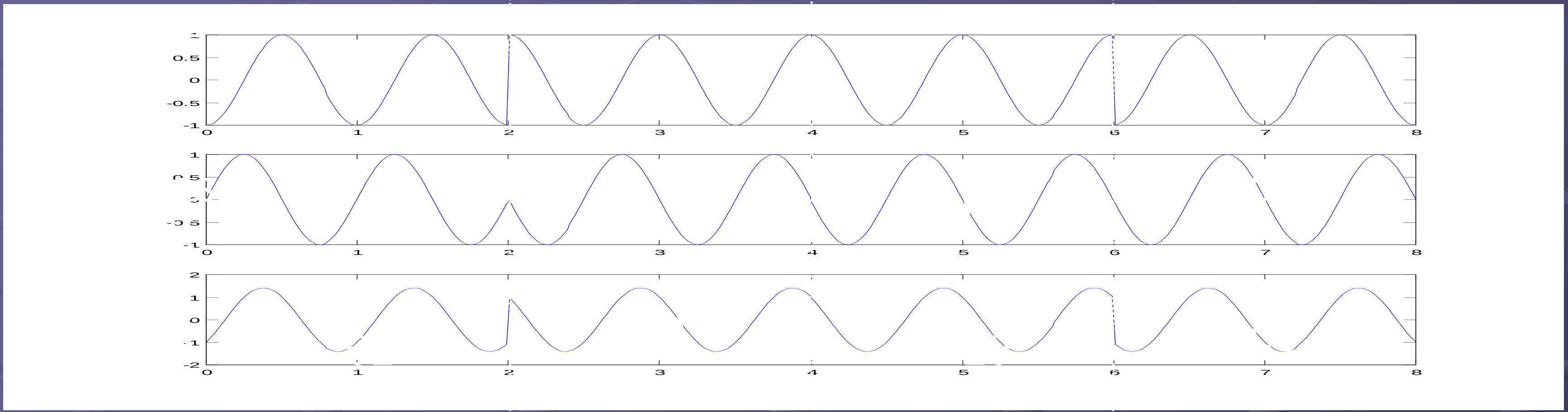
OFFSET QPSK

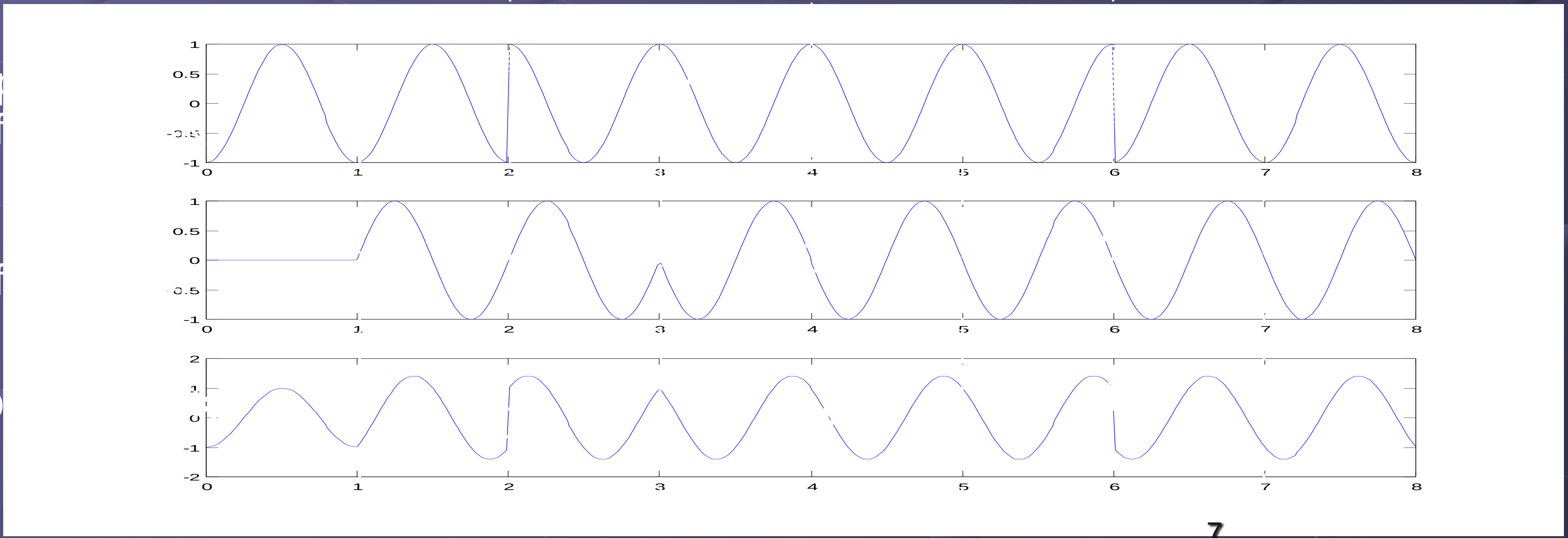
- To solve the amplitude fluctuation problem, we propose the offset QPSK.
- Offset QPSK delay the data in quadrature component by T/2 seconds (half of symbol).
- Now, no way that both bits can change at the same time.

OFFSET QPSK

- In the offset QPSK, the phase of the signal can change by ±90 or 0 degree only while in the QPSK the phase of the signal can change by ±180 ±90 or 0 degree.

OFFSET QPSK



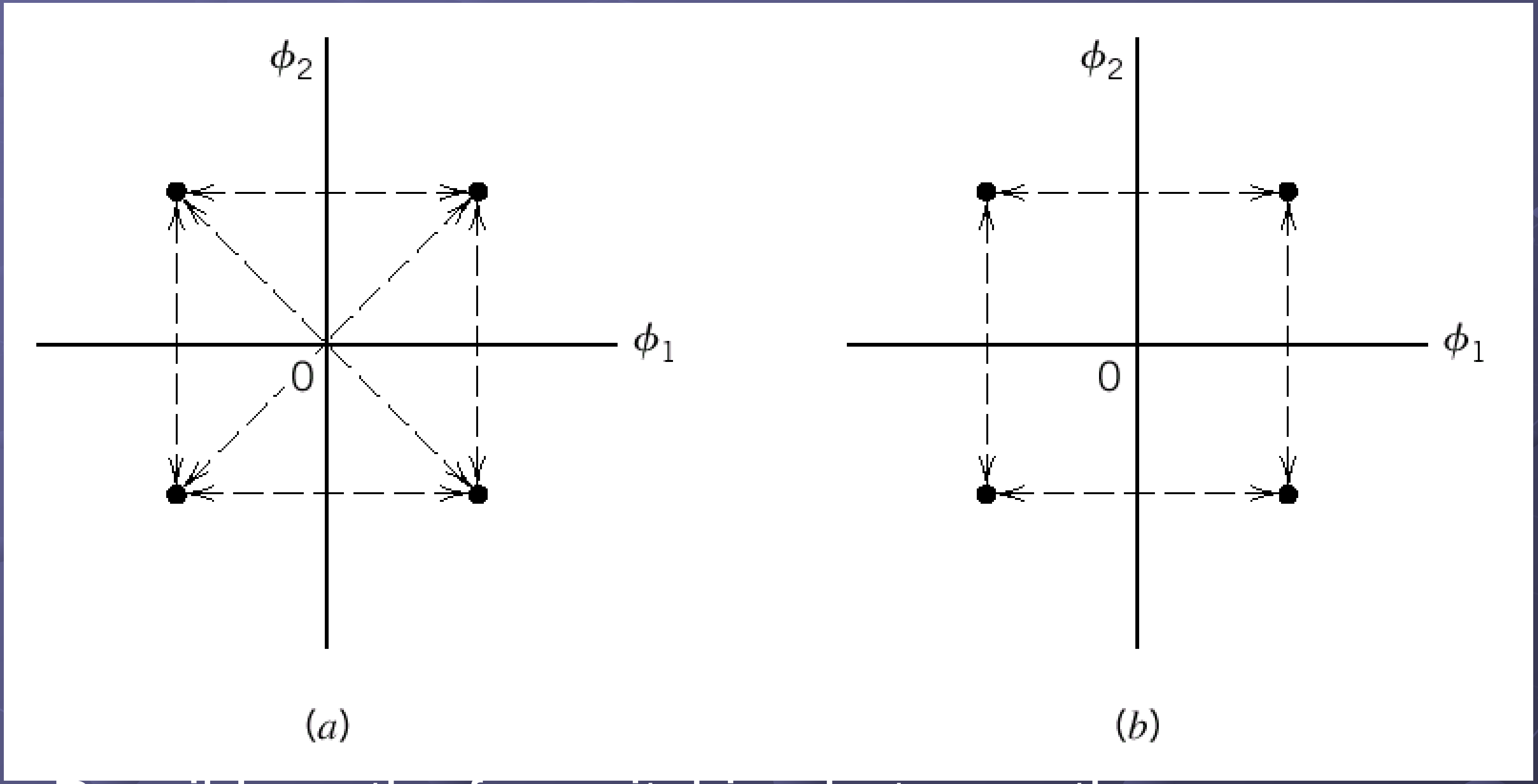


Inp
Of

Q
Of

O

Offset QPSK



Possible paths for switching between the message points in (*a*) QPSK and (*b*) offset QPSK.

OFFSET QPSK

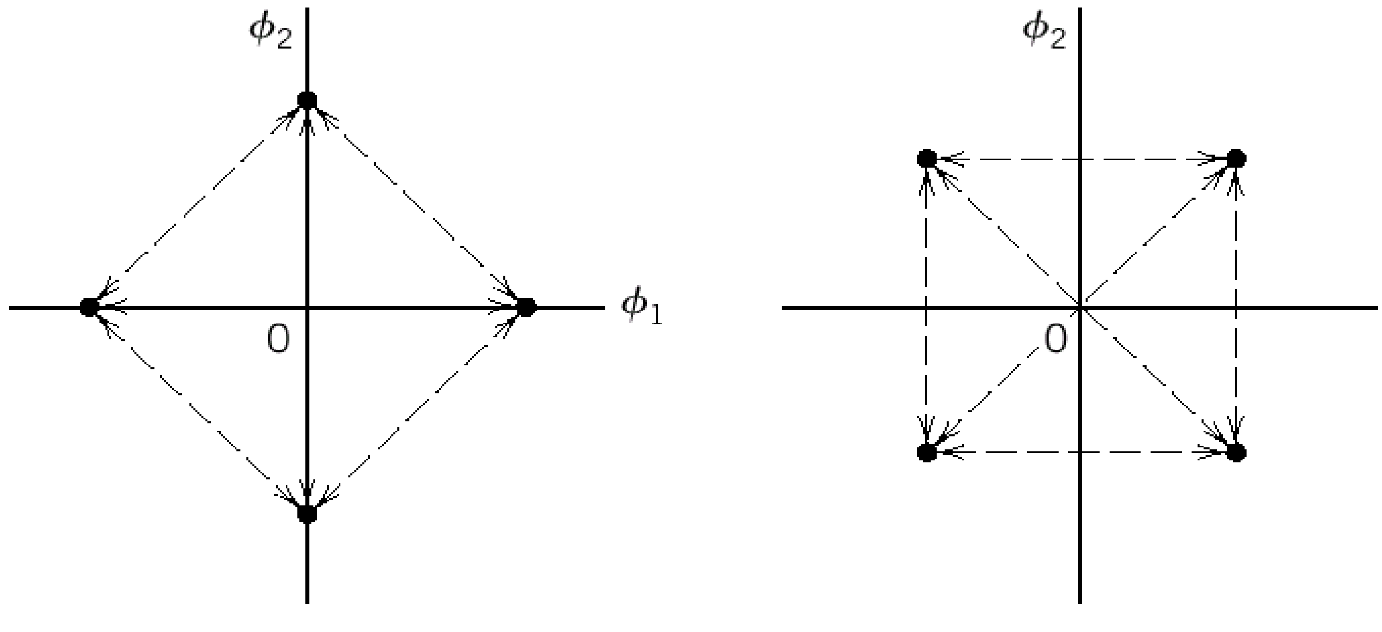
- Bandwidths of the offset QPSK and the regular QPSK is the same.
- From signal constellation we have that

$$P_e \approx \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

- Which is exactly the same as the regular QPSK.

π/4-shifted QPSK

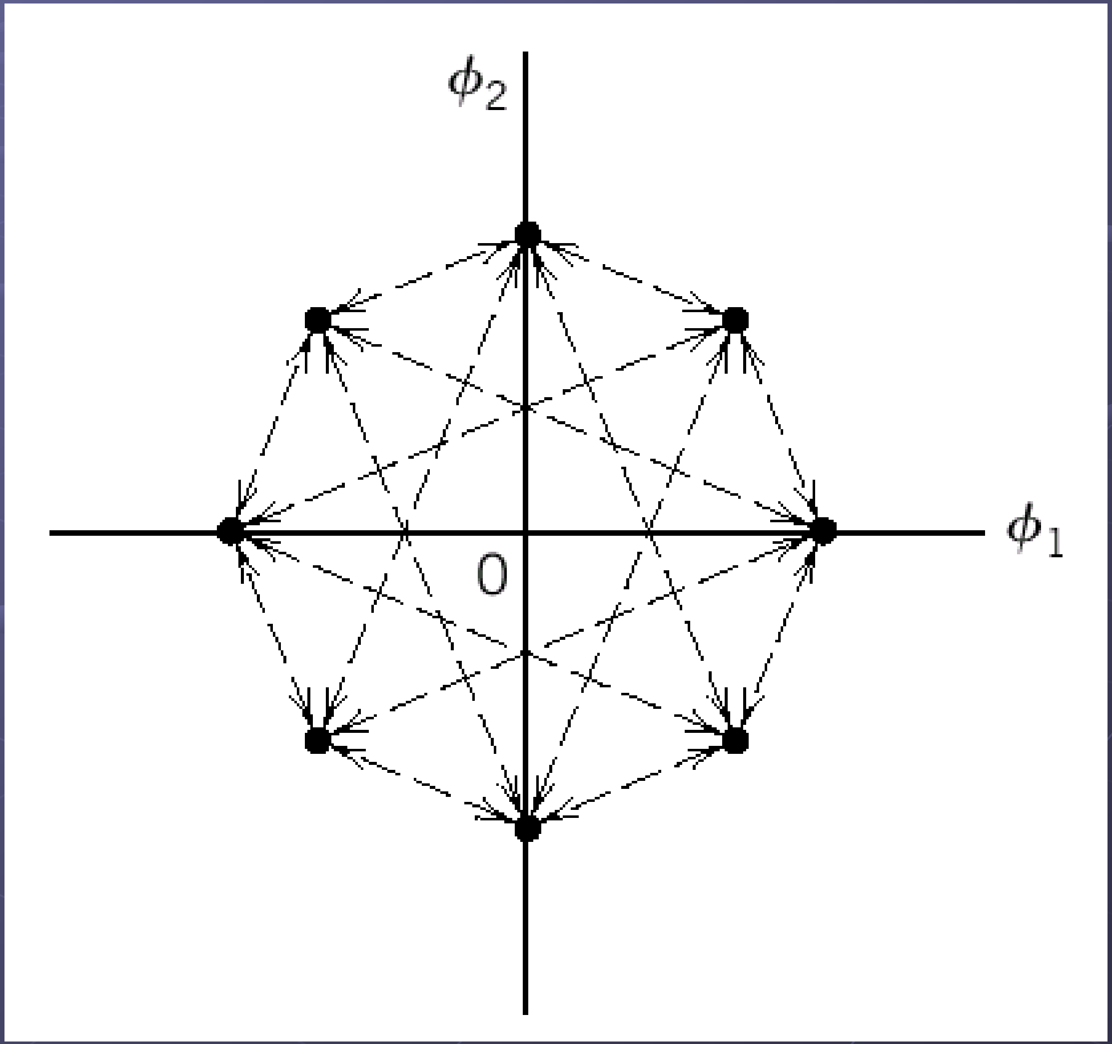
- Try to reduce amplitude fluctuation by switching between 2 signal constellation



(a) (b)

π/4-shifted QPSK

- As the result, the phase of the signal can be changed in order of ±π/4 or ±3π/4



$\pi/4$-shifted QPSK

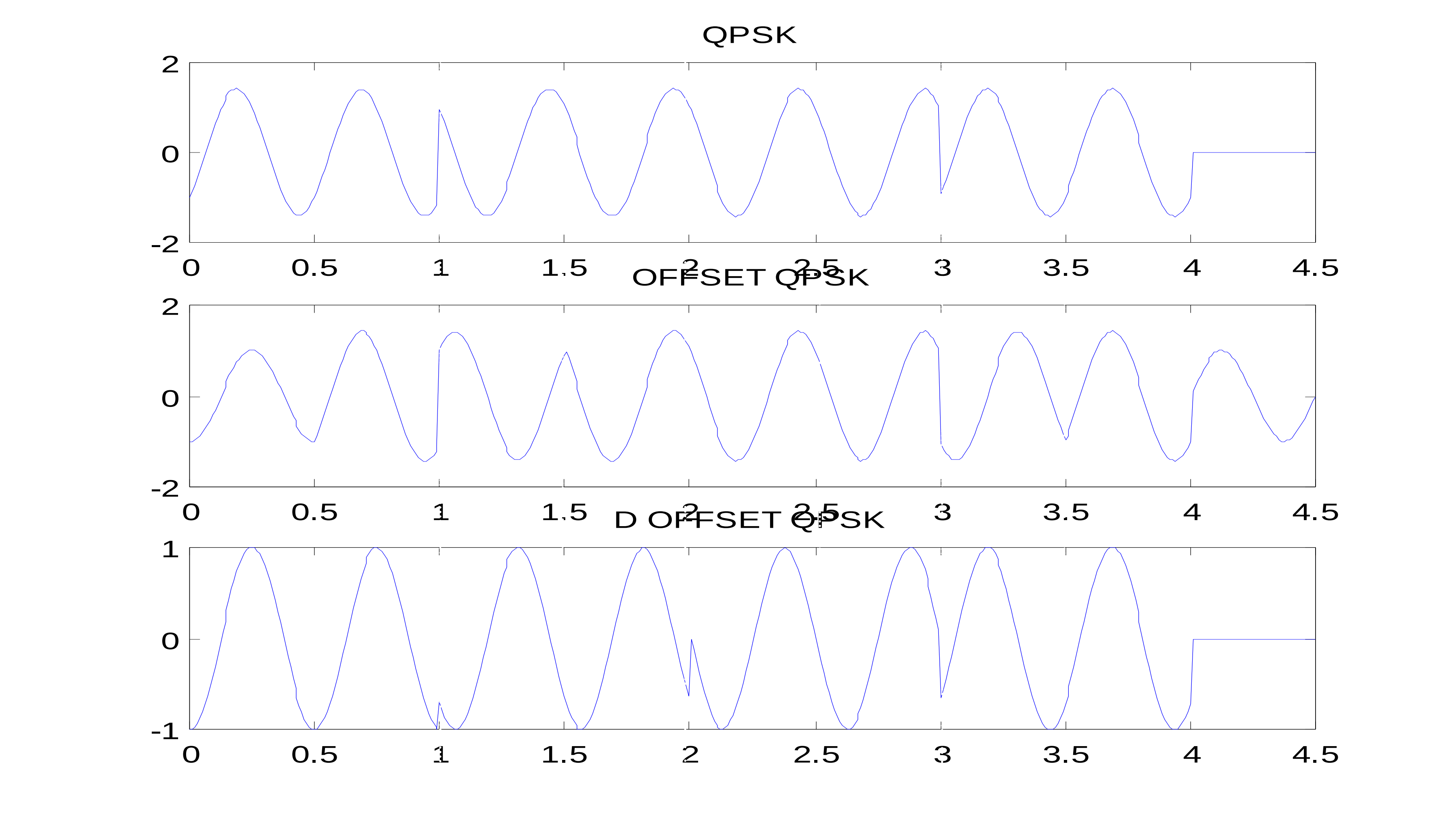
- Since the phase of the next will be varied in order of $\pm\pi/4$ and $\pm 3\pi/4$, we can designed the differential $\pi/4$-shifted QPSK as given below

Gray-Encoded Input Data	Phase Change in radians
00	$+\pi/4$
01	$+3\pi/4$
11	$-3\pi/4$
10	$-\pi/4$

$\pi/4$-shifted QPSK:00101001

Step	Initial phase	Input Dibit	Phase change	Transmitted phase
1	$\pi/4$	00	$\pi/4$	$\pi/2$
2	$\pi/2$	10	$-\pi/4$	$\pi/4$
3	$\pi/4$	10	$-\pi/4$	0
4	0	01	$3\pi/4$	$3\pi/4$

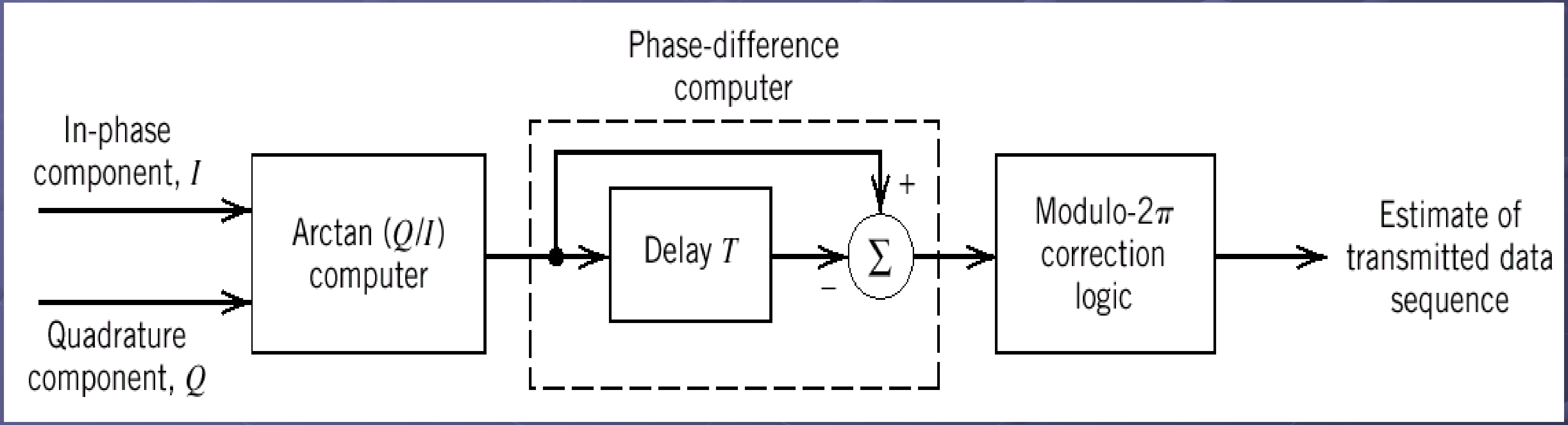
π/4-shifted QPSK



$\pi/4$-shifted QPSK

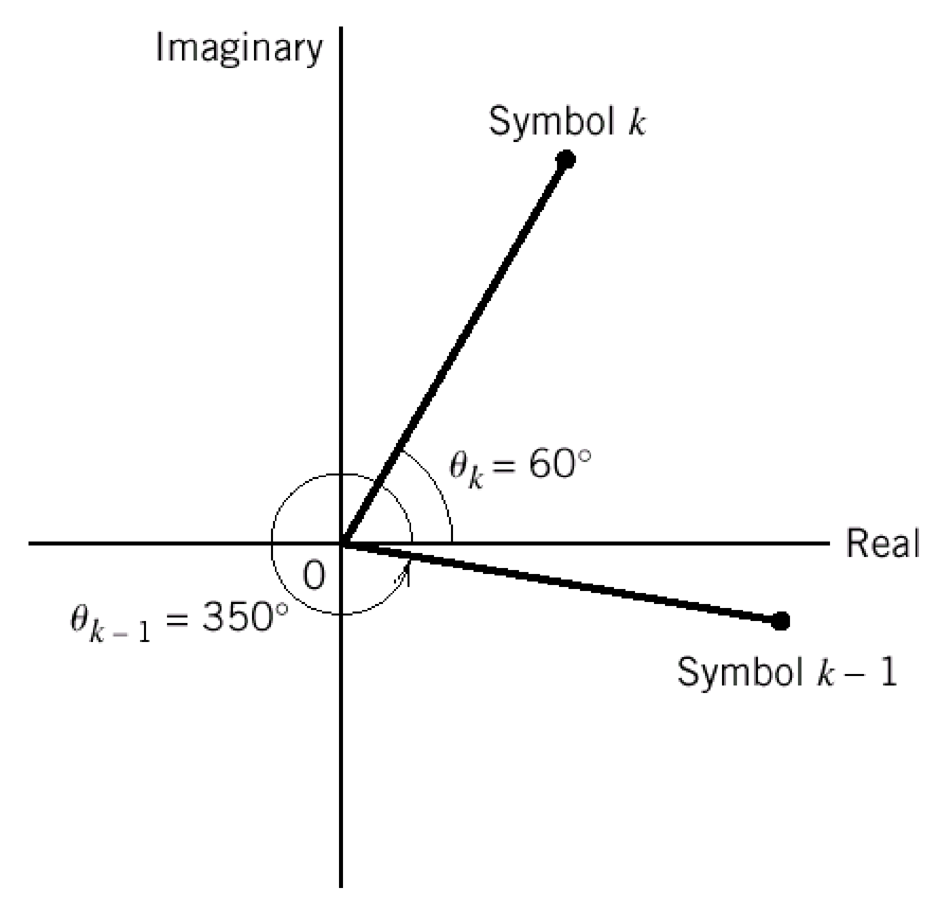
- Since we only measure the phase different between adjacent symbols, no phase information is necessary. Hence, non-coherent receiver can be used.

Block diagram of the $\pi/4$-shifted DQPSK detector.



π/4-shifted QPSK

- Illustrating the possibility of phase angles wrapping around the positive real axis.



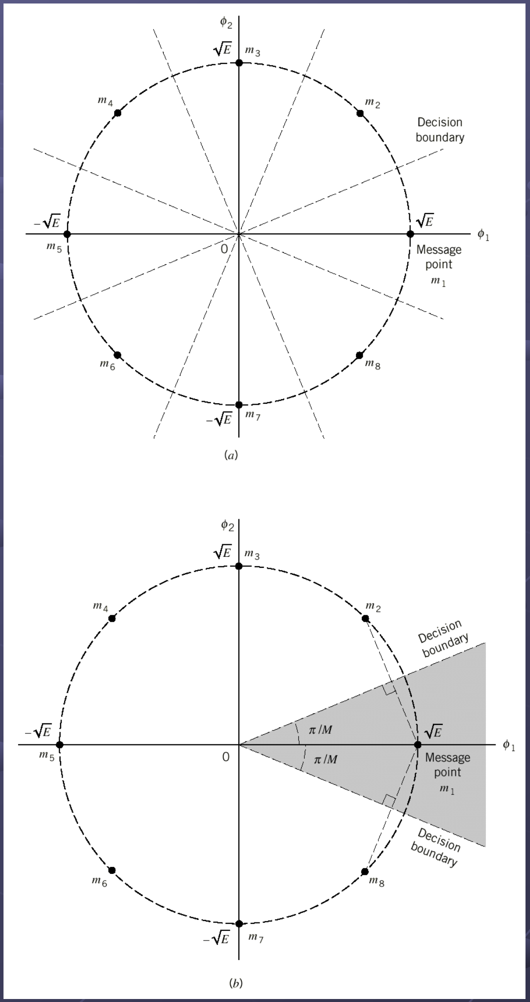
M-array PSK

- At a moment, there are M possible symbol values being sent for M different phase values, $\theta_i = 2(i-1)\pi / M$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1,2,\ldots,M$$

M-array PSK

- Signal-space diagram for octa phase-shift keying (i.e., $M = 8$). The decision boundaries are shown as dashed lines.
- Signal-space diagram illustrating the application of the union bound for octa phase shift keying.



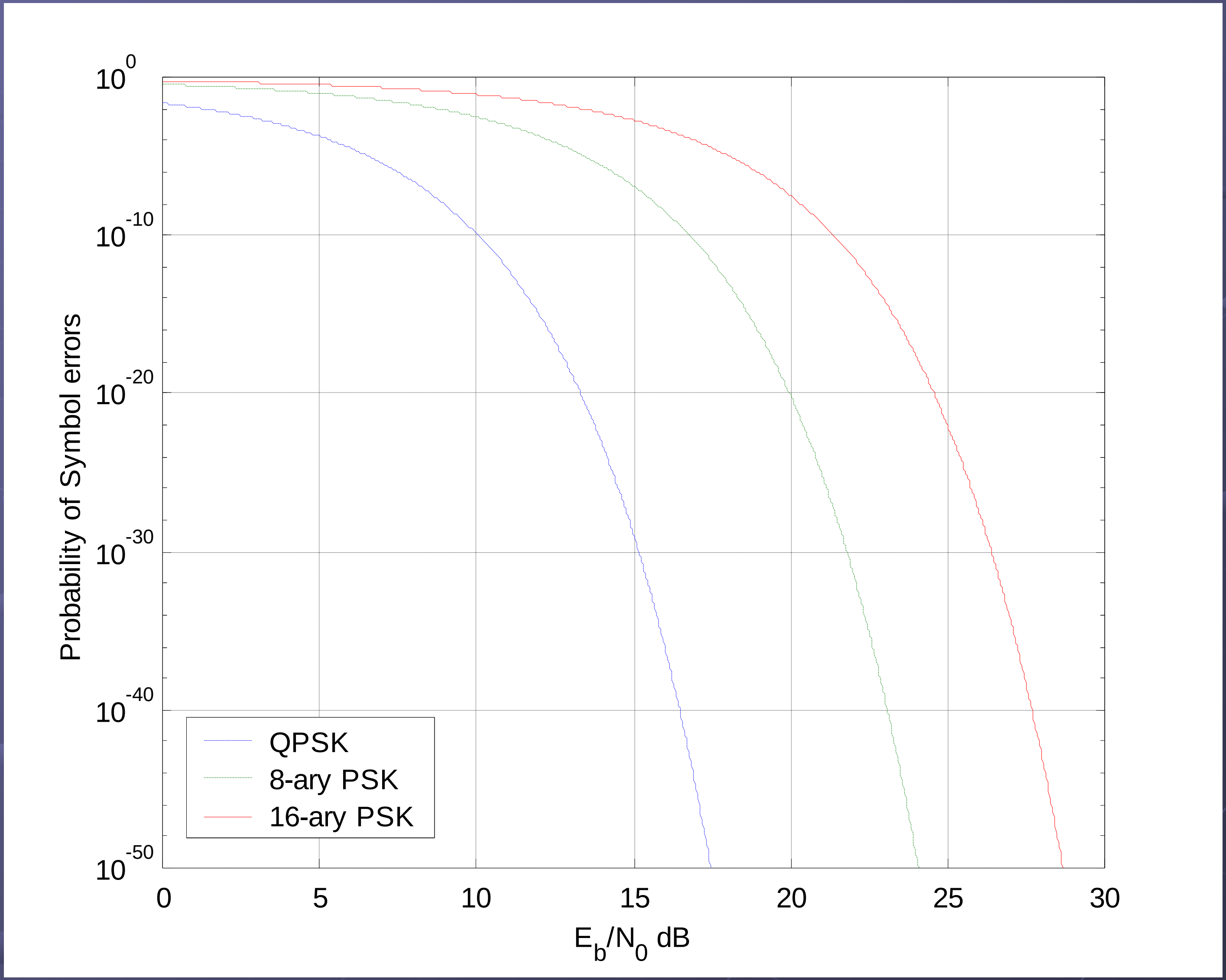
M-array PSK

- Probability of errors

$$\therefore d_{12} = d_{18} = 2\sqrt{E}\sin(\pi / M)$$

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}}\sin(\pi / M)\right); \quad M \geq 4$$

M-ary PSK



Probability of Symbol errors
10^{0}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
0
5
10
15
20
25
30
E_b/N_0 dB
QPSK
8-ary PSK
16-ary PSK

M-array PSK

- Power Spectra (M-array)

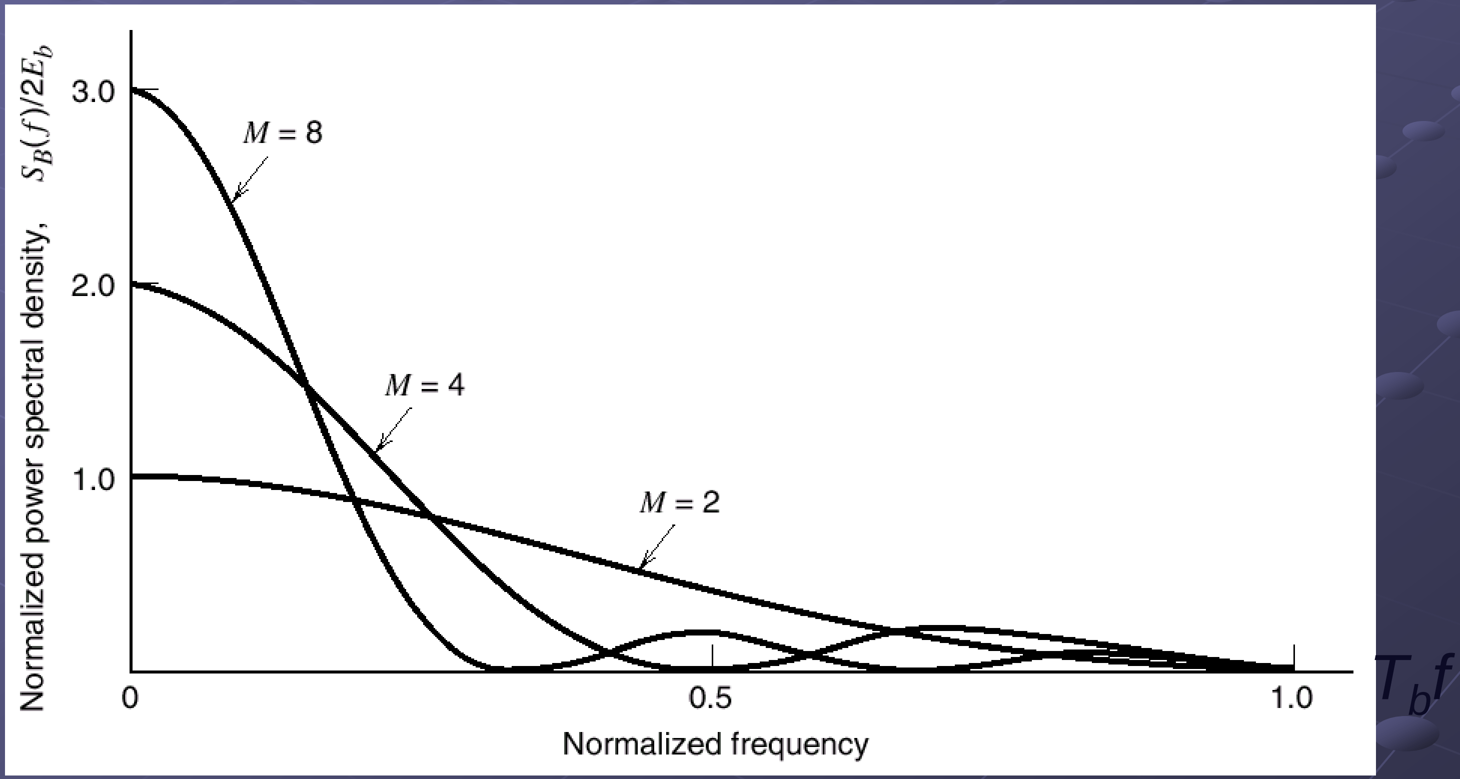
$$S_{PSK}(f) = 2E\,\mathrm{sinc}^2(Tf)$$
$$= 2E_b \log_2 M\,\mathrm{sinc}^2(T_b f \log_2 M)$$

- M=2, we have

$$S_{BPSK}(f) = 2E_b\,\mathrm{sinc}^2(T_b f)$$

M-array PSK

- Power spectra of *M*-ary PSK signals for M = 2, 4, 8.



M-array PSK

- Bandwidth efficiency:
  - We only consider the bandwidth of the main lobe (or null-to-null bandwidth)

$$B = \frac{2}{T} = \frac{2}{T_b \log_2 M} = \frac{2R_b}{\log_2 M}$$

  - Where B is the bandwidth of the transmitted data
  - Bandwidth efficiency of M-ary PSK is given by

$$\rho = \frac{R_b}{B} = \frac{R_b}{2R_b} \log_2 M = 0.5 \log_2 M$$

M-ary QAM

- QAM = Quadrature Amplitude Modulation
- Both Amplitude and phase of carrier change according to the transmitted symbol, m_i

$$s_i(t) = \sqrt{\frac{2E_0}{T}}a_i \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}b_i \sin(2\pi f_c t); \quad 0 < t \le T$$

where a_i and b_i are integers, E_0 is the energy of the signal with the lowest amplitude

M-ary QAM

- Again, we have

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \quad ;0 < t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t \quad ;0 < t \le T$$

as the basis functions

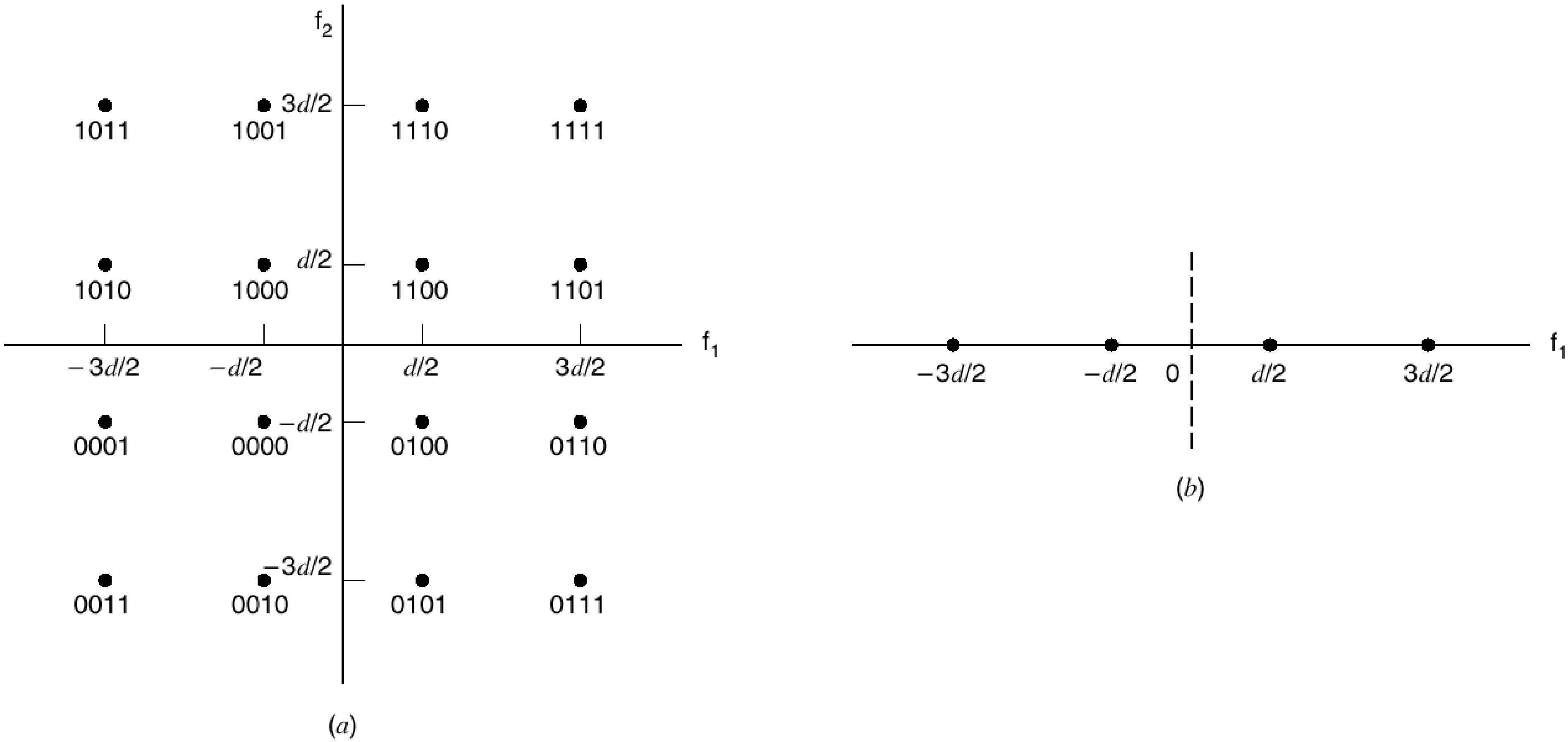
There are two QAM constellations, square constellation and rectangular constellation

M-ary QAM

- QAM square Constellation
  - Having even number of bits per symbol, denoted by $2n$
  - $M = L \times L$ possible values
  - Denoting $L = \sqrt{M}$

16-QAM

$$
\left[a_i , b_i\right] = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix}
$$



L-ary, 4-PAM

16-QAM

QAM Probability of error

- Calculation of Probability of errors
  - Since both basis functions are orthogonal, we can treat the 16-QAM as combination of two 4-ary PAM systems.
  - For each system, the probability of error is given by

$$P_e' = \left(1 - \frac{1}{L}\right) erfc\left(\frac{d}{2\sqrt{N_0}}\right) = \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{E_0}{N_0}}\right)$$

16-QAM Probability of error

- A symbol will be received correctly if data transmitted on both 4-ary PAM systems are received correctly. Hence, we have

$$P_c(symbol) = (1 - P_e')^2$$

- Probability of symbol error is given by

$$P_e(symbol) = 1 - P_c(symbol) = 1 - (1 - P_e')^2$$
$$= 1 - 1 + 2P_e' - (P_e')^2 \approx 2P_e'$$

16-QAM Probability of error

- Hence, we have

$$P_e(symbol) = 2\left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{E_0}{N_0}}\right)$$

- But because average energy is given by

$$E_{av} = 2\left[\frac{2E_0}{L}\sum_{i=1}^{L/2}(2i-1)^2\right] = \frac{2(M-1)E_0}{3}$$

- We have

$$P_e(symbol) = 2\left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$