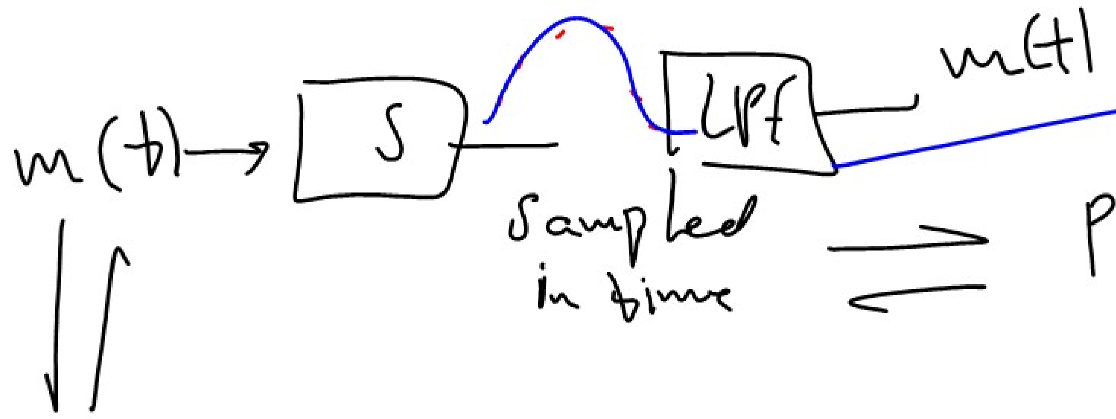
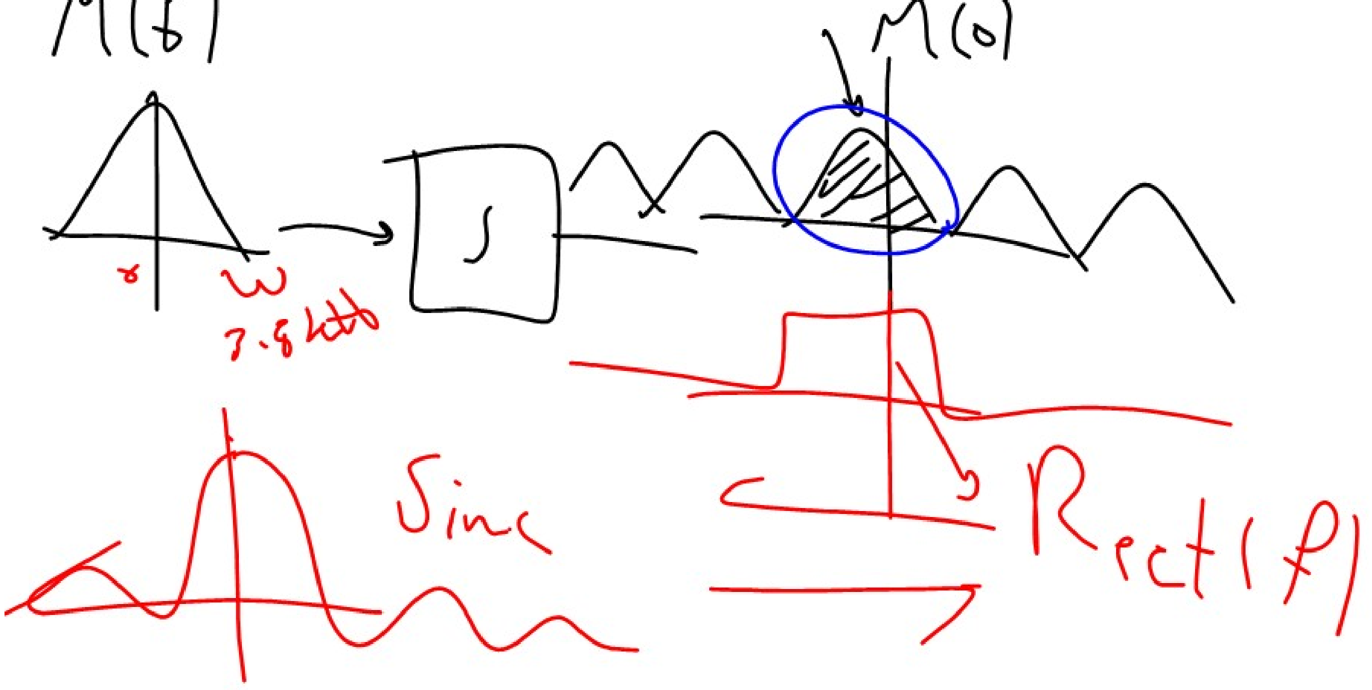
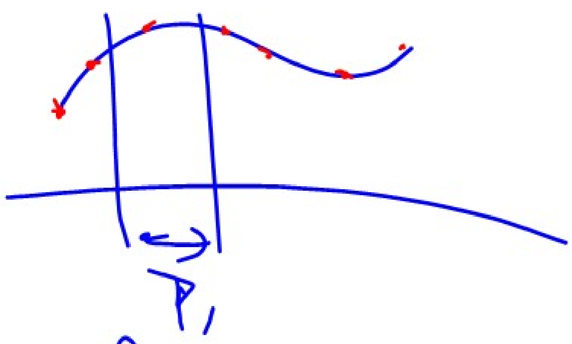


→ Synthesis

sampled in time ⇌ periodic in freq

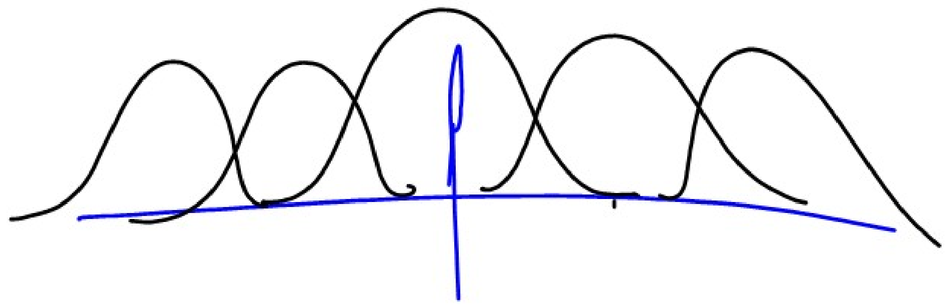




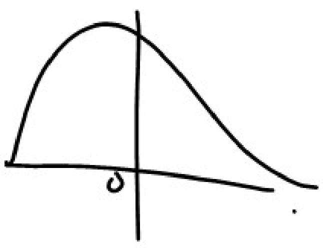
Nyquist thereom

$f_s \geq 2W$

$f_s = \frac{1}{T_s}$



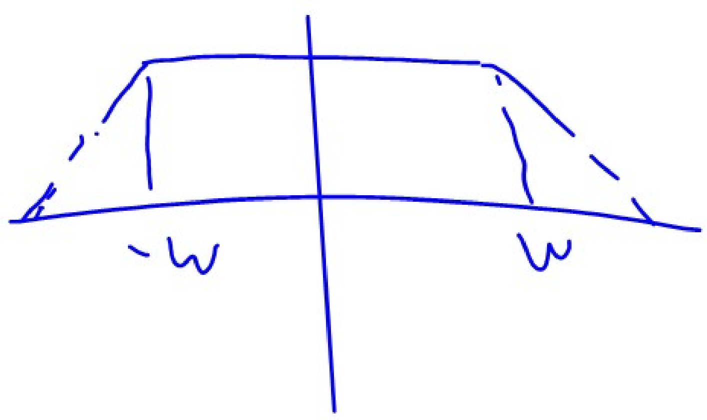
if $f_s < 2W$

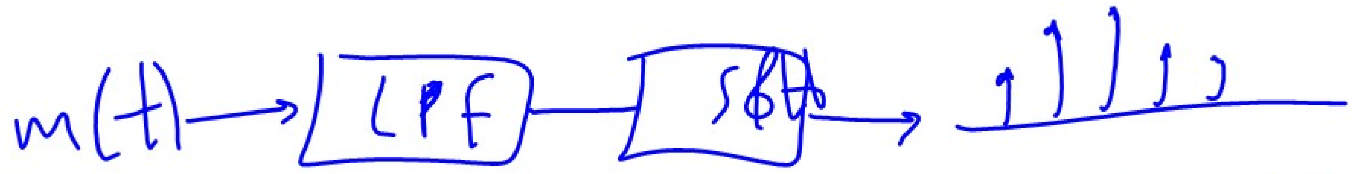


aliasing

→ anti-aliasing filter

$f_s \geq 2W$





$f_s \geq 2W$

pulse time Mod

PTM

AM

FM

PAM

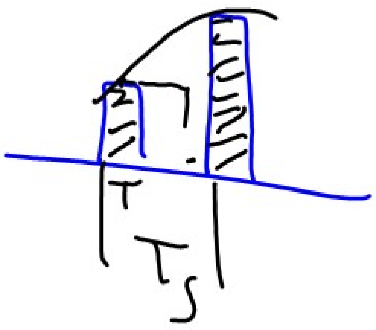
$BW \neq 0$

pulse Amp Mod (PAM)

pulse Width Mod (PWM)

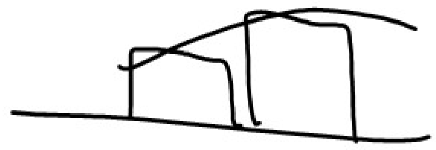
pulse Position Mod (PPM)

$BW = \frac{1}{T} = \frac{1}{0} = \infty$



$T < T_s$

$\frac{T}{T_s} < 0.1$

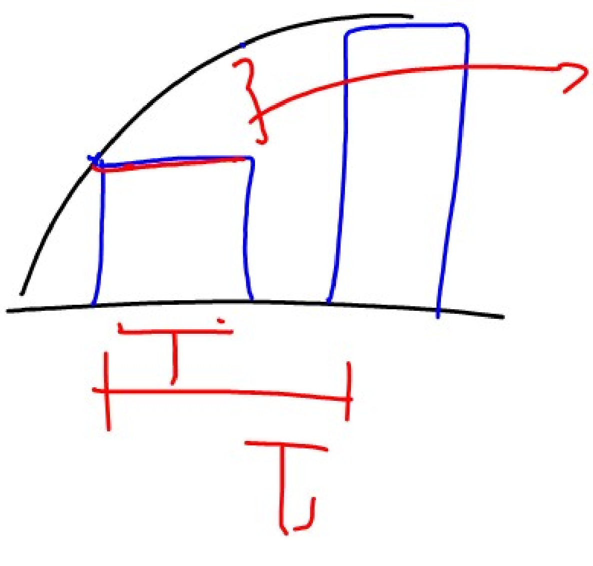


Rect()

Sampling:

flat-top Sampler

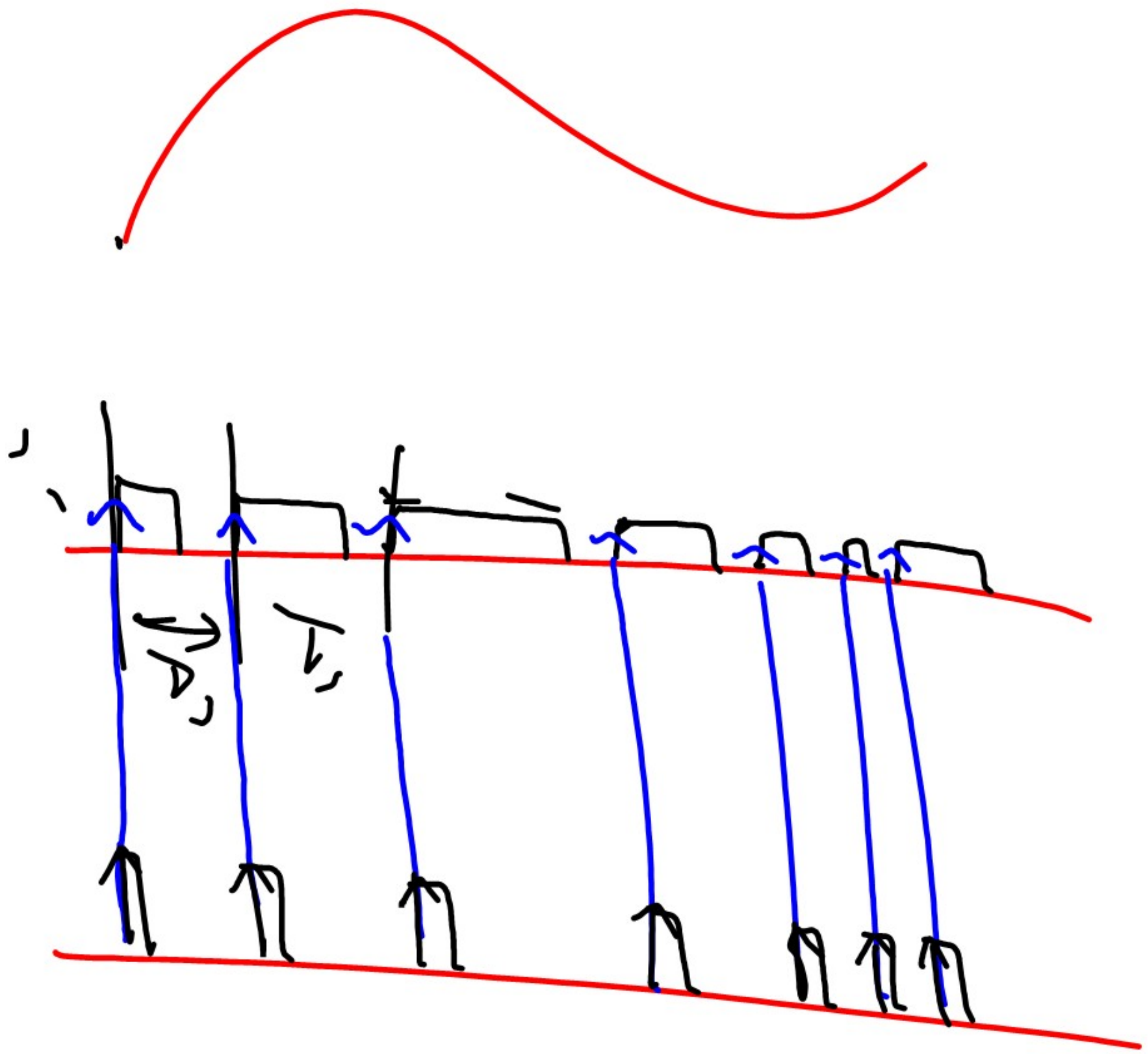
natural Sampler



ap.arture effect

$\frac{T}{T_s} < 0.1$

PWM



Control

PWM

PPM

Communications and Signals Processing

Dr. Ahmed Masri

Department of Communications

An Najah National University

2012/2013

Chapter 5 - Outlines

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

- In ***continuous-wave (CW)*** modulation, which we studied in Chapters 3 and 4, some parameter of a sinusoidal carrier wave is varied ***continuously*** in accordance with the message signal
- This is in direct contrast to ***pulse modulation***, which we study in the present chapter
- In ***pulse modulation***, some parameter of a pulse train is varied in accordance with the message signal

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

In the first step from analog to digital:

- An analog source is sampled at discrete times
- The resulting analog samples are then transmitted by means of analog pulse modulation.
  - ***Pulse-Amplitude Modulation (PAM),*** the simplest form of analog pulse modulation.
  - ***Pulse-Position Modulation (PPM)***

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

In the second step from analog to digital:

- An analog source is not only sampled at discrete times but also the samples themselves are also ***quantized to discrete levels***.
  - *Pulse-code Modulation (PCM)*
  - *Delta Modulation (DM)*
  - *Differential pulse-code modulation*

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

Advantages of digital transmission over analog transmission:

- Digital systems are less sensitive to noise than analog.
  - For long transmission lengths, the signal may be regenerated effectively error-free at different point along the path and the original signal transmitted over the remaining length.

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

With digital systems, it is easier to integrate different services, for example,

- Video and the accompanying soundtrack, into the same transmission scheme.

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

The transmission scheme can be relatively independent of the source. For example

- A digital transmission scheme that transmits voice at 10 kbps could also be used to transmit computer data at 10 kbps

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

Circuitry for handling digital signals is easier to repeat and digital circuits are less sensitive to physical effect such as vibration and temperature

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

Digital signals are simpler to characterize and typically ***do not have the same amplitude range and variability as analog signals***.

- This makes the associated hardware easier to design.

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

Digital techniques offer strategies for more efficient use of media, e.g. cable, radio wave, and optical fibers.

- Various media sharing strategies, known as ***multiplexing techniques***, are more easily implemented with digital transmission strategies

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

There are techniques <u>for removing redundancy</u> from a digital transmission, so as to minimize the amount of information that has to be transmitted.

These techniques fall under the broad classification of ***source coding*** and some of these techniques are discussed in Chapter 10

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

There are techniques for adding controlled redundancy to digital transmission, such that errors occur during transmission may be corrected at the receiver without any additional information.

These techniques fall under the general category of ***channel coding***, which is described in Chapter 10.

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

Digital techniques make it easier to specify complex standards that may be shared on a worldwide basis.

This allows the development of communication components with many different features (e.g., a cellular handset) and their interoperation with a different component (e.g., a base station) produced by a different manufacturer

Introduction – *Pulse Modulation: Transition From Analog To Digital Communications*

Why Digitize Analog Source ?

It should be emphasized that the majority of these advantages for digital transmission rely ***on availability of low-cost microelectronics***

Section 5.1 – *Sampling Process*

Section 5.1 – *Sampling Process*

- Through use of the sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.
- Recall that the Fourier transform of a ***periodic signal*** with period $\mathbf{T_0}$ consists of an infinite sequence of delta functions occurring at integer multiples of the fundamental frequency $\mathbf{f_0 = 1/T_0}$

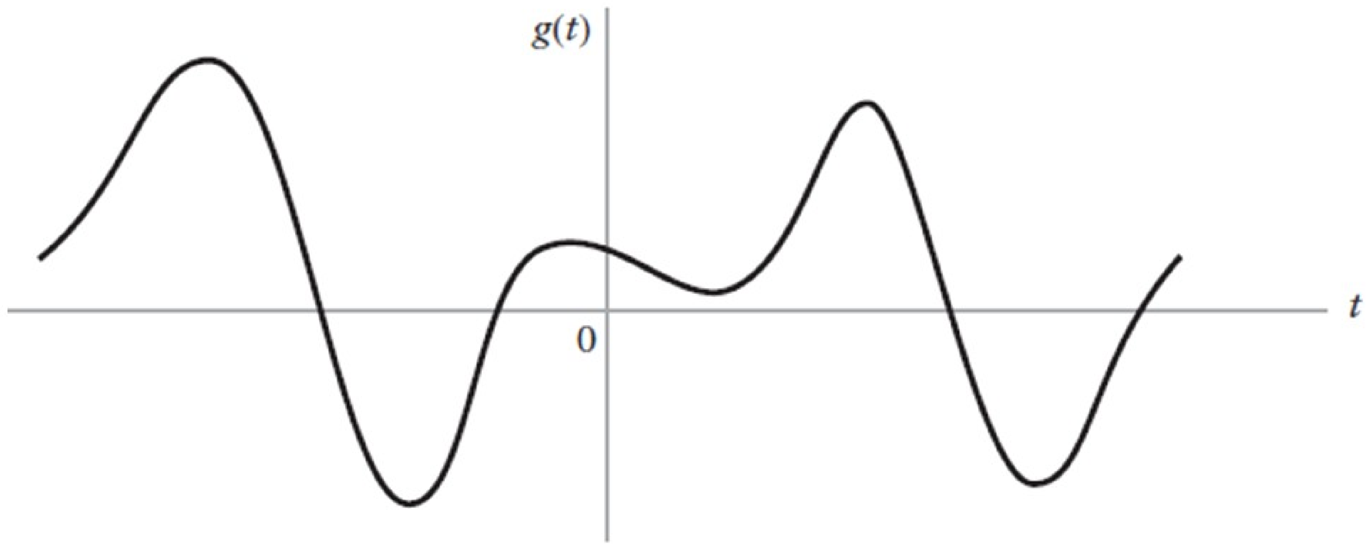
Section 5.1 – *Sampling Process*

- We may therefore state that:
  - ***Making a signal periodic in the time domain has the effect of sampling the spectrum of the signal in the frequency domain***
  - Using the duality property of the Fourier transform, ***we state that sampling a signal in the time domain has the effect of making the spectrum of the signal periodic in the frequency domain***
- This latter issue is the subject of this section

Section 5.1 – *Sampling Process*

Sampling process: an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time

- It is necessary that we choose the ***sampling rate*** properly, so that the sequence of samples ***uniquely*** defines the original analog signal
- Consider an arbitrary signal g(t) of finite energy, which is specified for all time t



(a)

Dr. Ahmed Masri

Section 5.1 – *Sampling Process*

- Suppose that we sample the signal g(t) instantaneously and at a uniform rate, once every T_s seconds
- We obtain an infinite sequence of samples spaced T_s seconds a part and denoted by $\{\mathbf{g(nT_s)}\}$
  - where **n** takes on all possible integer values, both positive and negative

Section 5.1 – *Sampling Process*

- Let $\mathbf{g_\delta(t)}$ denote the ideal sampled signal, obtained by individually weighting the elements of a periodic sequence of **Dirac delta** functions spaced T_s seconds apart by the sequence of numbers $\{\mathbf{g(nT_s)}\}$, as given by

$$g_\delta(t) = g(t)\cdot\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \qquad (5.1)$$

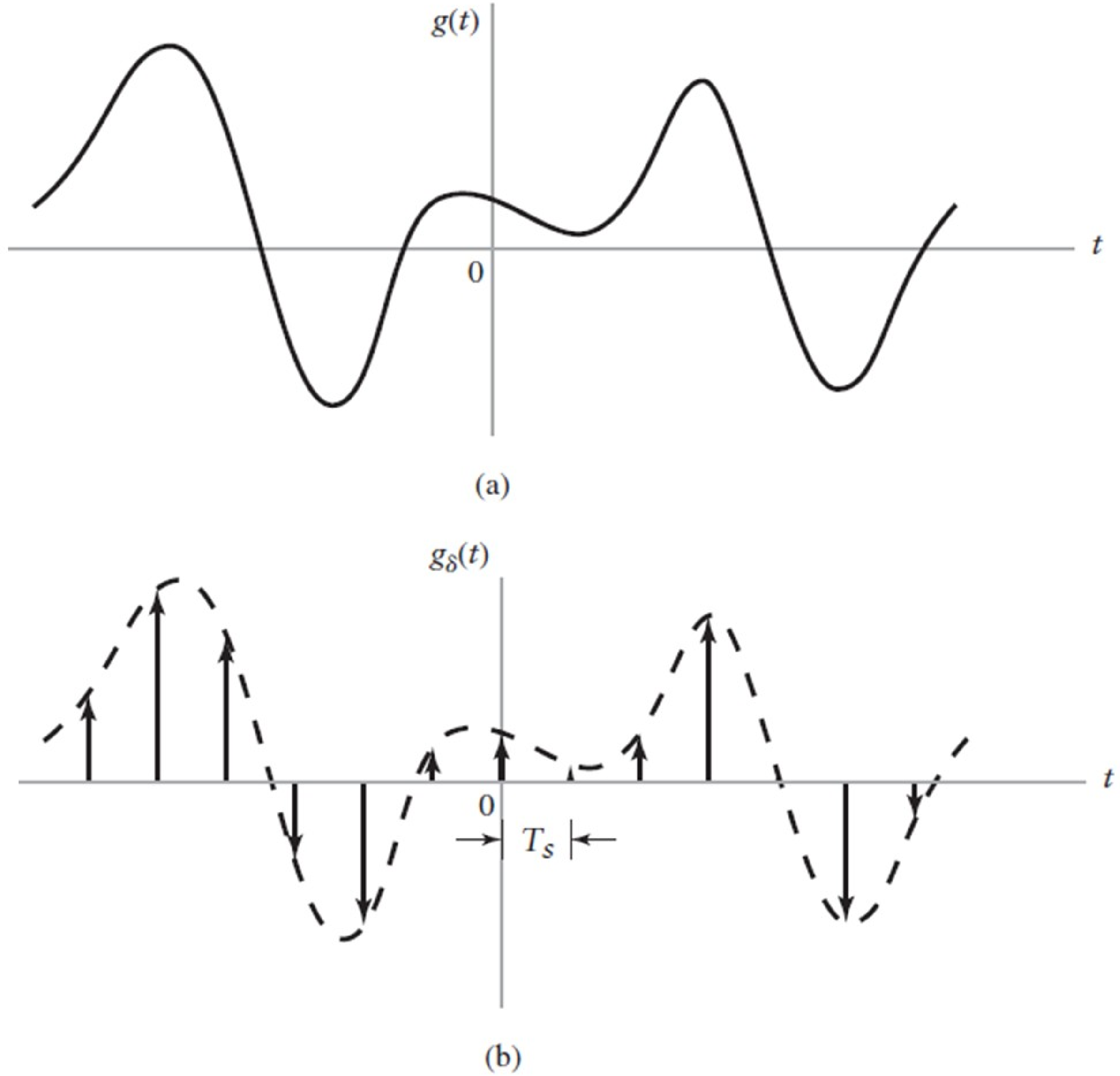
- We refer to $\mathbf{g_\delta(t)}$ as ***the instantaneously (ideal) sampled signal***

Section 5.1 – *Sampling Process*

- A delta function weighted in this manner is closely approximated by ***a rectangular pulse of duration Δt and amplitude $g(nT_s)/\Delta t$***, the smaller **Δt** we make the better the approximation will be

Section 5.1 – *Sampling Process*

We refer to T_s as the ***sampling period*** and $f_s = 1/T_s$ as the ***sampling rate***



Section 5.1 – *Sampling Process*

- The instantaneously sampled signal **$g_\delta(t)$** has a mathematical form similar to that of the ***Fourier transform of a periodic signal.***

$$\sum_{m=-\infty}^{\infty} g(t - mT_0) \rightleftharpoons f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0) \qquad (2.88)$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \qquad (\mathbf{5}.1)$$

- We may determine the Fourier transform of the sampled signal **$g_\delta(t)$** by invoking the duality property of the Fourier transform

Section 5.1 – *Sampling Process*

TABLE 5.1 *Time-Frequency Sampling-Duality Relationships*

Ideal sampling in the frequency domain (Discrete spectrum); see Chapter 2	*Ideal sampling in the time domain (Discrete-time function); see this chapter*
Fundamental period $T_0 = 1/f_0$	Sampling rate $f_s = 1/T_s$
Delta function $\delta(f - mf_0)$, where $m = 0, \pm 1, \pm 2, \ldots$	Delta function $\delta(t - nT_s)$ where $n = 0, \pm 1, \pm 2, \ldots$
Periodicity in the time-domain	Periodicity in the frequency domain
Time-limited function	Band-limited spectrum
$T_0 \sum_{m=-\infty}^{\infty} g(t - mT_0) = \sum_{n=-\infty}^{\infty} G(nf_0) e^{j2\pi n f_0 t}$ $\rightleftharpoons$ $\sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0)$	$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$ $\rightleftharpoons$ $\sum_{n=-\infty}^{\infty} g(nT_s) e^{-j2\pi n T_s f} = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$

Section 5.1 – *Sampling Process*

Instantaneous Sampling and Frequency-Domain Consequences

Applying the duality in the table, we get the Fourier transform

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \qquad (5.1)$$

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \qquad \mathbf{(5.2)}$$

where ***G(f)*** is the Fourier transform of the original signal **g(t)** and f_s is the sampling rate $1/T_s$.

Section 5.1 – *Sampling Process*

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \qquad \textbf{(5.2)}$$

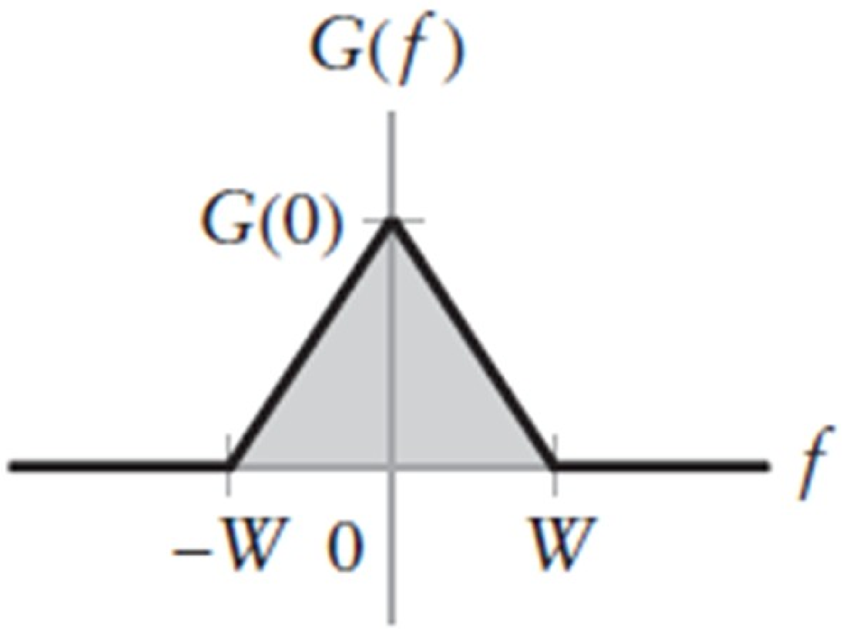
Eq. (5.2) state that the process of uniformly sampling a continuous times signal of finite energy results in a periodic spectrum with a period equal to the sampling rate

Section 5.1 – *Sampling Process*

Sampling Theorem

For a strictly band-limited g(t), with no frequency components higher than W Hz, then

$$G(f) = 0 \quad \text{for } |f| \geq W \qquad \text{(Band-Limited Signal)}$$



(a)

Section 5.1 – *Sampling Process*

Sampling Theorem

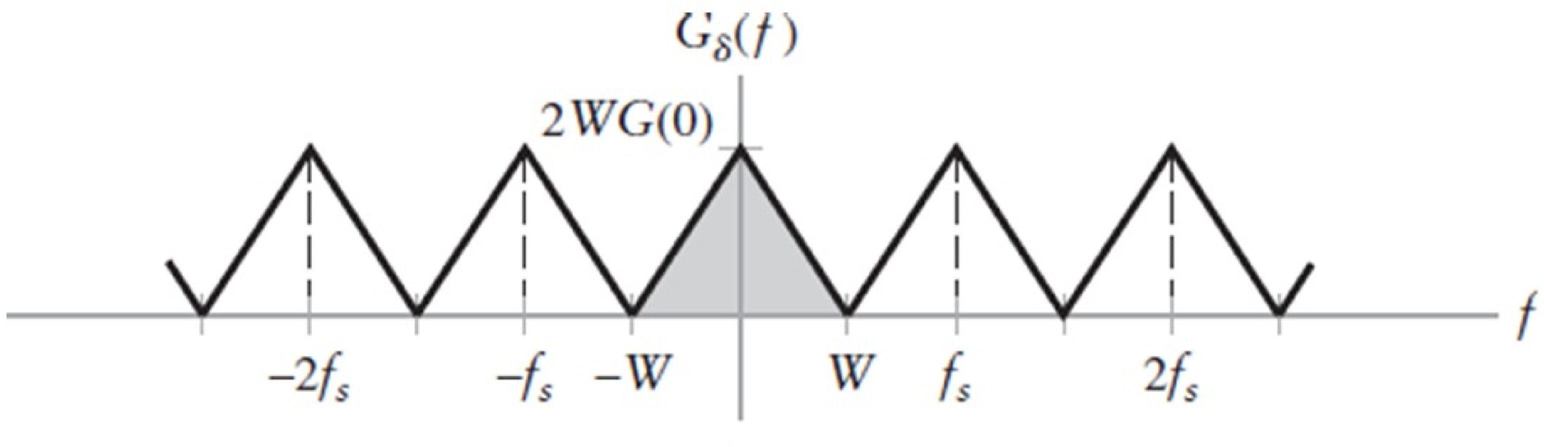
◊ Using a sampling period $\mathbf{T_s = ½ W}$, then

$$f_s = 2W \left(\text{or } T_s = \frac{1}{2W} \right)$$

The spectrum $G_\delta(f)$ of the sampled signal $g_\delta(t)$ is

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\exp(-j2\pi nfT_s) \qquad (5.3)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right) \qquad (5.4)$$



(b)

Dr. Ahmed Masri

Section 5.1 – *Sampling Process*

From Eq. (5.2), we readily see that the Fourier transform of $\mathbf{g_\delta(t)}$ may also be expressed as

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \qquad \textbf{(5.2)}$$

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s) \qquad (5.5)$$

For a strictly band-limited signal and under the two conditions

$$G_\delta(f) = 2W\,G(f)\,, \quad -W < f < W$$

$$G(f) = \frac{1}{2W} G_\delta(f), \quad -W < f < W \qquad (5.6)$$

Section 5.1 – *Sampling Process*

From Eq. (5.5 and 5.6)

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s)$$

$$G(f) = \frac{1}{2W} G_\delta(f), \quad -W < f < W$$

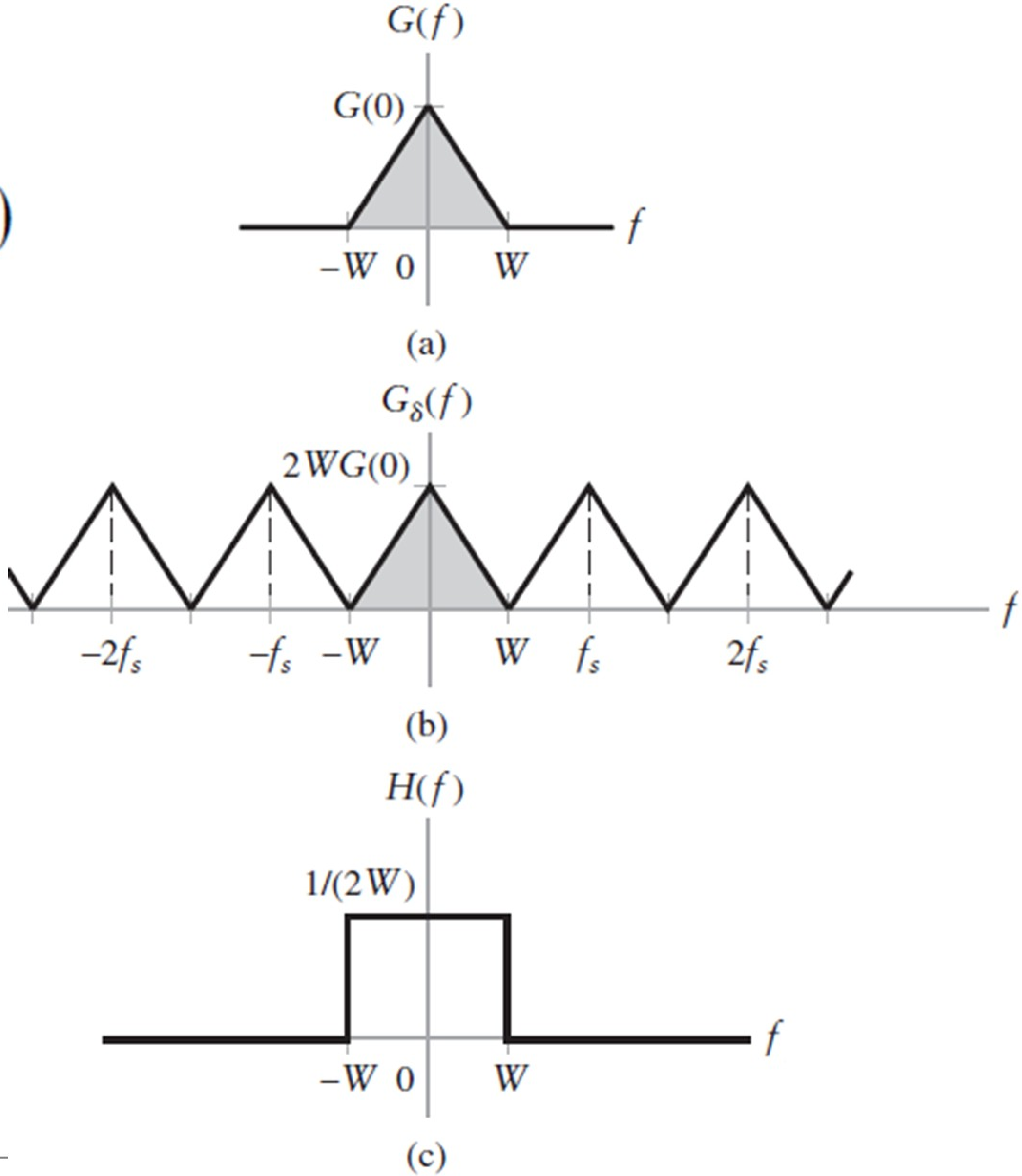


FIGURE 5.2 (*a*) Spectrum of a strictly band-limited signal $g(t)$. (*b*) Spectrum of instantaneously sampled version of $g(t)$ for a sampling period $T_s = 1/2W$. (*c*) Frequency response of ideal low-pass filter aimed at recovering the original message signal $g(t)$ from its uniformly sampled version.

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right) \qquad (5.4)$$

◊ Substituting Eq. (5.4) in Eq. (5.6), we may also write

$$G(f) = \frac{1}{2W} G_\delta(f), \quad -W < f < W$$

$$G(f) = \frac{1}{2W}\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right), \quad -W < f < W \qquad (5.7)$$

◊ Therefore, if the sample values of a signal g(t) are specified for all time, then the Fourier transform G(f) of the signal is <u>uniquely determined</u> by using the discrete time Fourier transform of Eq. (5.7) .

◊ In the other words, the sequence {g(n/2W)} has all the information contained in g(t).

Section 5.1 – *Sampling Process*

Reconstructing the signal of g(t)

Substituting Eq. (5.7) in the formula for the <u>inverse Fourier transform</u> g(t) in terms of G(f), we get

$$g(t) = \int_{-\infty}^{\infty} G(f)\ \exp(j2\pi ft)\,df$$

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \exp(j2\pi ft)\,df$$

Section 5.1 – *Sampling Process*

Reconstructing the signal of g(t)

◊ Interchanging the order of summation and integration

$$g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{1}{2W}\int_{-W}^{W}\exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]df \qquad (5.8)$$

$$g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{\sin(2\pi Wt-n\pi)}{(2\pi Wt-n\pi)}$$

$$g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\operatorname{sinc}(2Wt-n) \quad -\infty<t<\infty \qquad (5.9)$$

Eq. is the <u>interpolation formula</u> for reconstructing the original signal g(t) from the sequence of sample values {g(n/2W)}

Section 5.1 – *Sampling Process*

Reconstructing the signal of g(t)

◊ *Interpolation* is a method of constructing new data points within the range of ***a discrete set*** of known data points

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\operatorname{sinc}(2Wt-n) \quad -\infty < t < \infty \qquad (5.9)$$

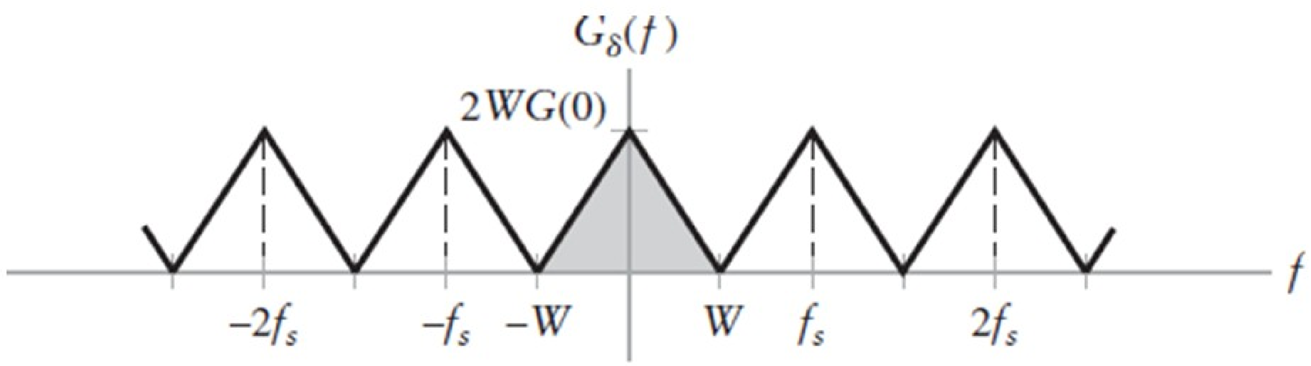
◊ Each sample is multiplied by a delayed version of the interpolation function, and all the resulting waveforms are added to obtain g(t)

Section 5.1 – *Sampling Process*

Reconstructing the signal of g(t)

◊ In light of Eq. (5.4): The *synthesis filter* or *reconstruction* filter aimed at recovering the original strictly band-limited signal **g(t)** from its instantaneously sampled version $\mathbf{g_\delta(t)}$ in accordance with Eq. (5.7) *consists of an ideal low-pass* whose frequency response is limited exactly to the same band as the signal **g(t)**

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right) \quad (5.4)$$



(b)

Section 5.1 – *Sampling Process*

We may now state the ***sampling theorem*** for strictly band-limited signals of finite energy in two equivalent parts:

1. ***Analysis***.(Transmitter part) A band-limited signal of finite energy that has no frequency components higher than W hertz is completely ***described*** by specifying the values of the signal at instants of time separated by 1/2W seconds.

Section 5.1 – *Sampling Process*

2. ***Synthesis***.(Receiver part) A band-limited signal of finite energy that has no frequency components higher than W hertz is completely ***recovered*** from knowledge of its samples taken at the rate of 2W samples per second.

Section 5.1 – *Sampling Process*

- The sampling rate of 2W samples per second for a signal bandwidth of W hertz is called the ***Nyquist rate***;
- And its reciprocal 1/2W (measured in seconds) is called the ***Nyquist interval***
- ***Note also that the Nyquist rate is the minimum sampling rate permissible.***

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - I

- Derivation of the sampling theorem is based on the assumption that the signal g(t) is strictly band-limited
- In practice, however, no information-bearing signal of physical origin is strictly band-limited
- As a result, some degree of ***under sampling*** is always encountered

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - II

- Consequently, ***aliasing*** is produced by the sampling process
- ***Aliasing*** refers to the phenomenon of a high-frequency component in the spectrum of the signal seemingly taking on the identity of a lower frequency in the spectrum of its sampled version, as illustrated in Fig. 5.3

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - III

- Fig. 5.3

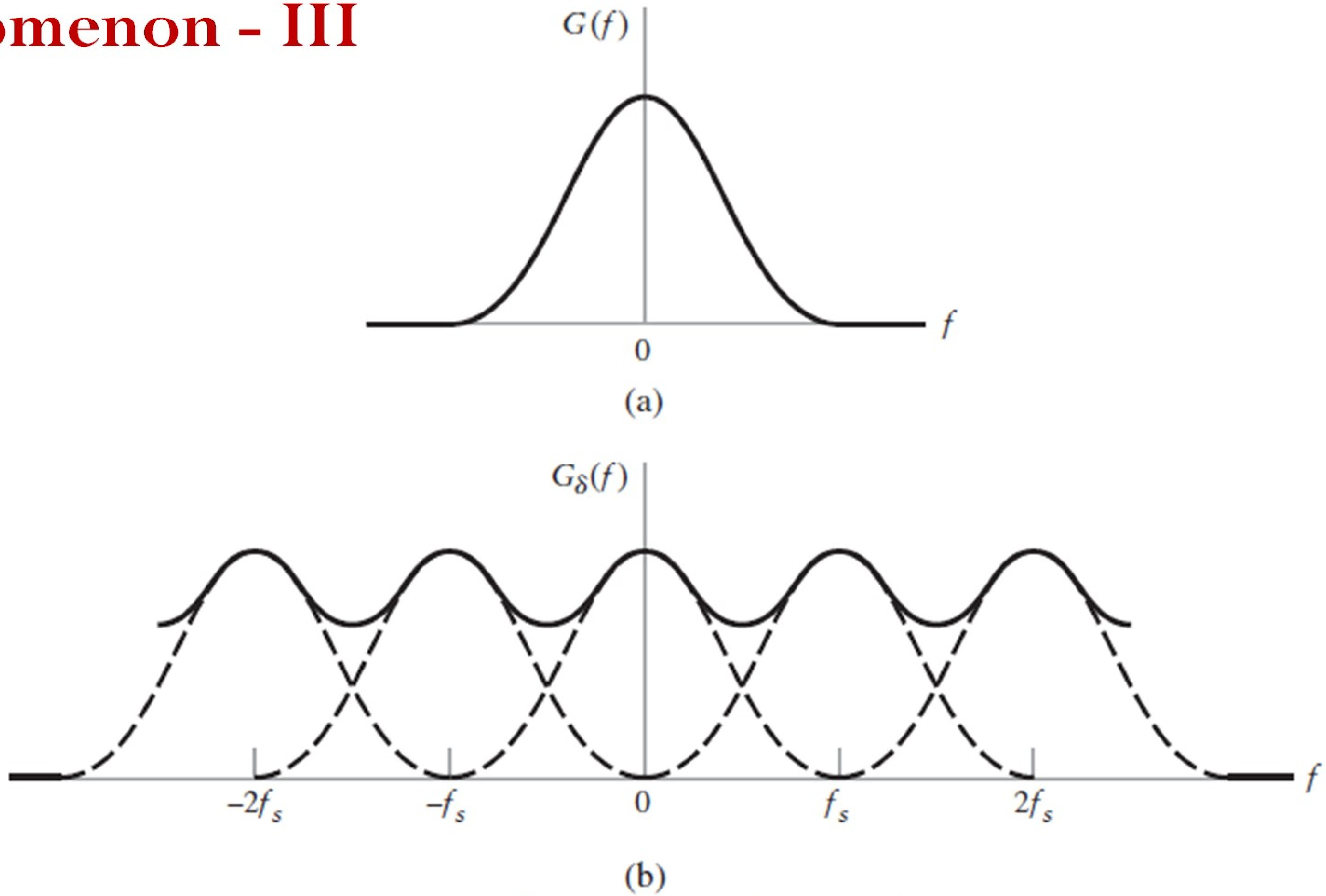


FIGURE 5.3 (*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal, exhibiting the aliasing phenomenon.

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - IV

- The aliased spectrum shown by the solid curve in Fig. 5.3(b) pertains to an "***undersampled***" version of the message signal represented by the spectrum of Fig. 5.3(a).

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - V

To combat the effects of aliasing in practice, we may use two corrective measures:

1. Prior to sampling, a ***low-pass anti-alias filter*** is used to attenuate those high-frequency components of a message signal that are not essential to the information being conveyed by the signal
2. The filtered signal is sampled at ***a rate slightly higher than the Nyquist rate***

Section 5.1 – *Sampling Process*

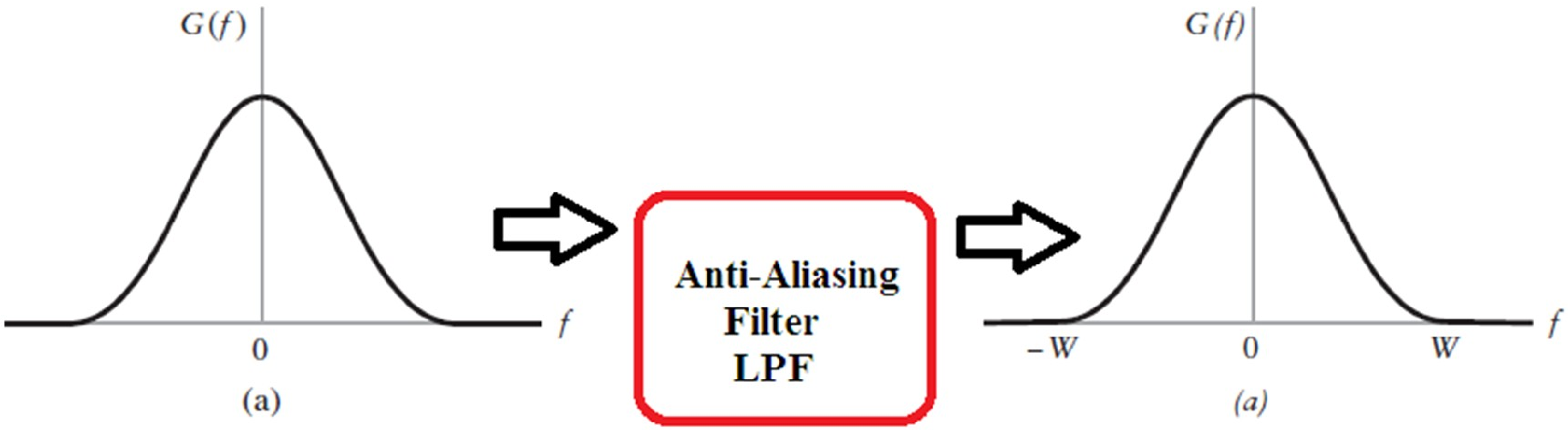
Aliasing Phenomenon - VI

- The use of a sampling rate higher than the Nyquist rate also has the beneficial effect of easing the design of the ***synthesis filter*** used to recover the original signal from its sampled version

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - VII

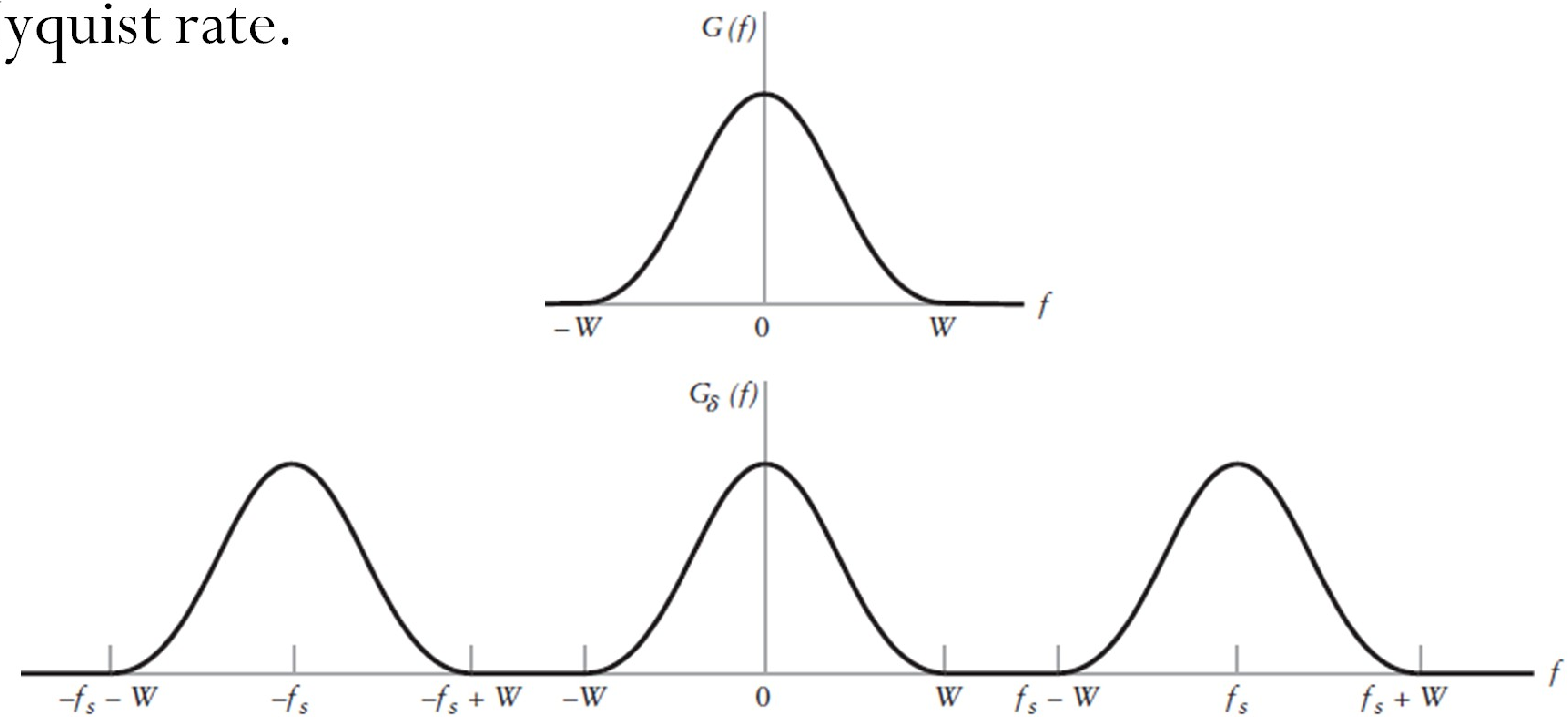
Consider the example of a message signal that has been anti-alias (low-pass) filtered, resulting in the spectrum shown in Fig. 5.4(a)



Section 5.1 – *Sampling Process*

Aliasing Phenomenon - VIII

The spectrum of the instantaneously sampled version of the signal is shown in Fig. 5.4(b), assuming a sampling rate higher than the Nyquist rate.

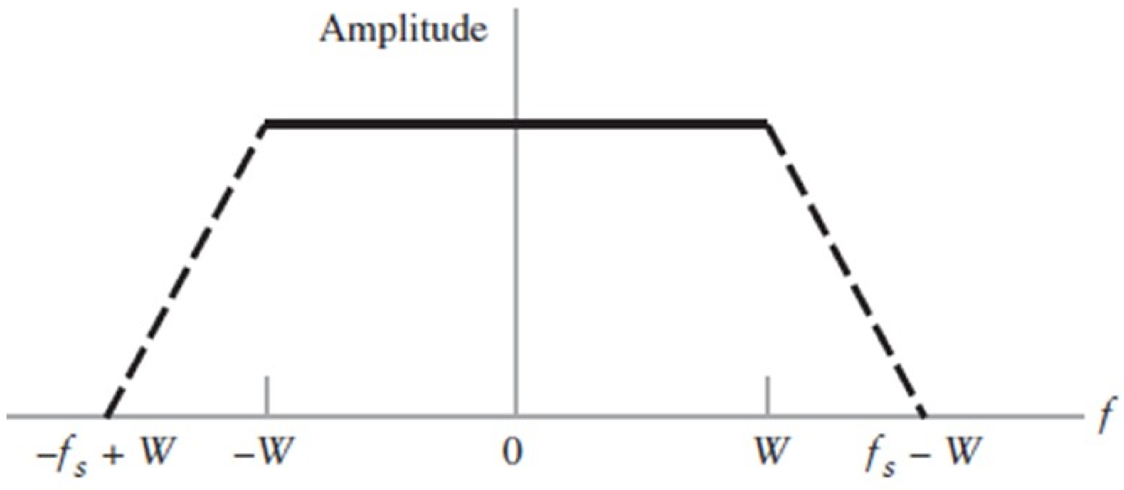


(b)

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - IX

The design of a physically realizable ***synthesis filter***, may be achieved as follows (see Fig. 5.4(c)):

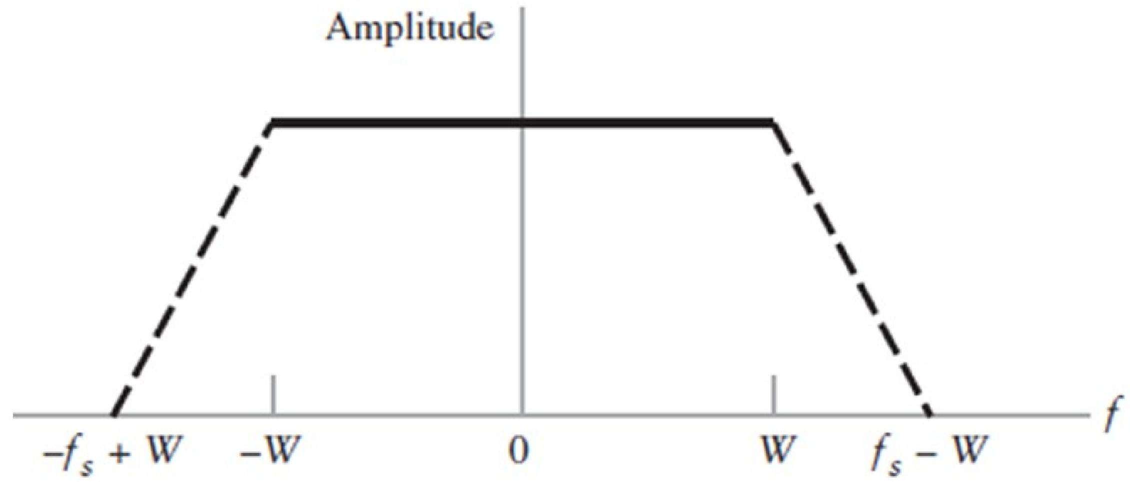


(c)

1. The reconstruction filter is of a low-pass kind with a passband extending from –W to W, which is itself determined by the anti-alias filter

Section 5.1 – *Sampling Process*

Aliasing Phenomenon - IX



(c)

2. The filter has a non-zero transition band extending (for positive frequencies) from W to f_s –W where f_s is the sampling rate.

Section 5.2 – Pulse-Amplitude Modulation

Section 5.2 –Pulse-Amplitude Modulation

The simplest and most basic form of analog pulse modulation techniques

- In ***Pulse-Amplitude Modulation (PAM),*** the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal

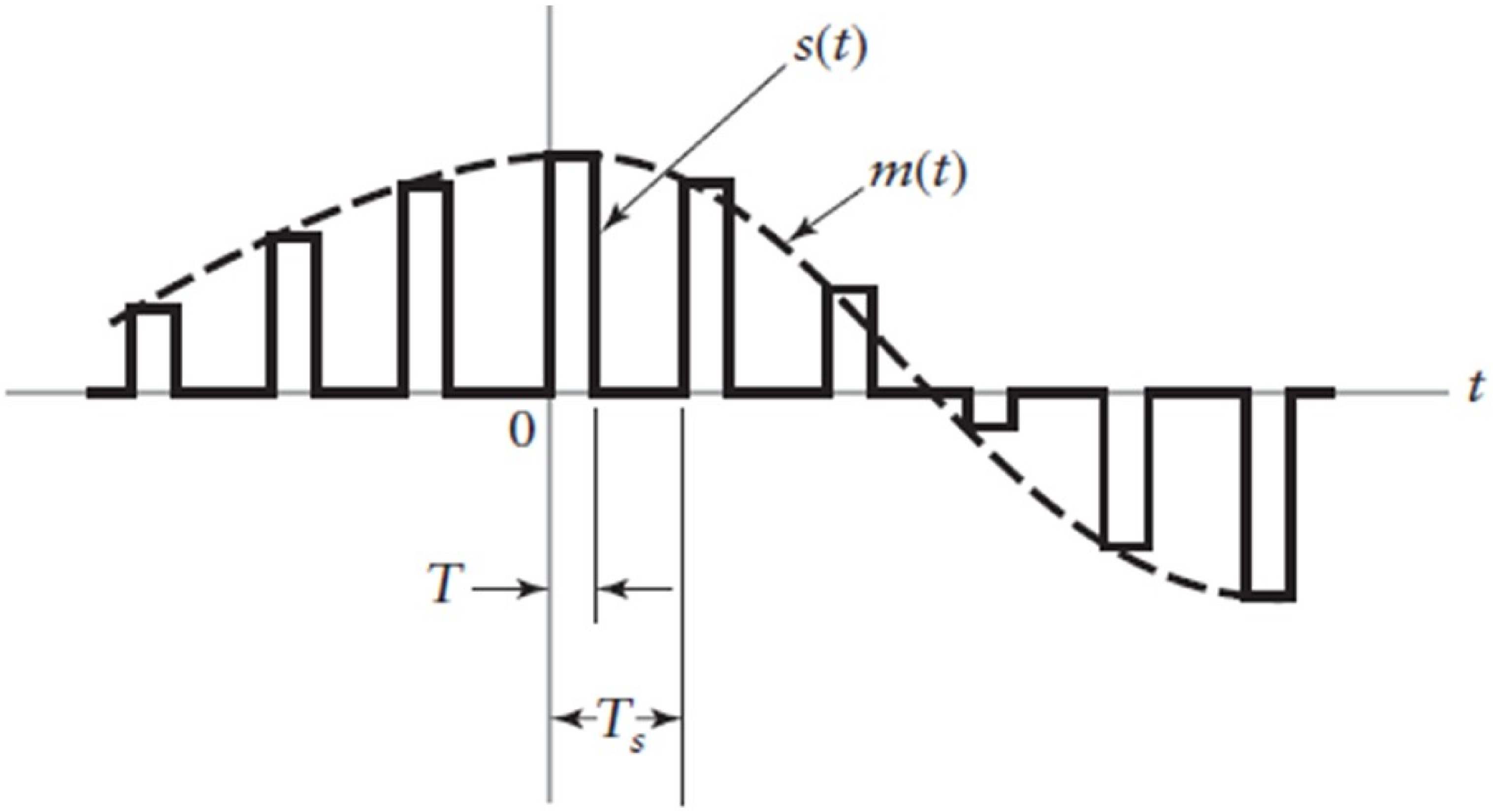
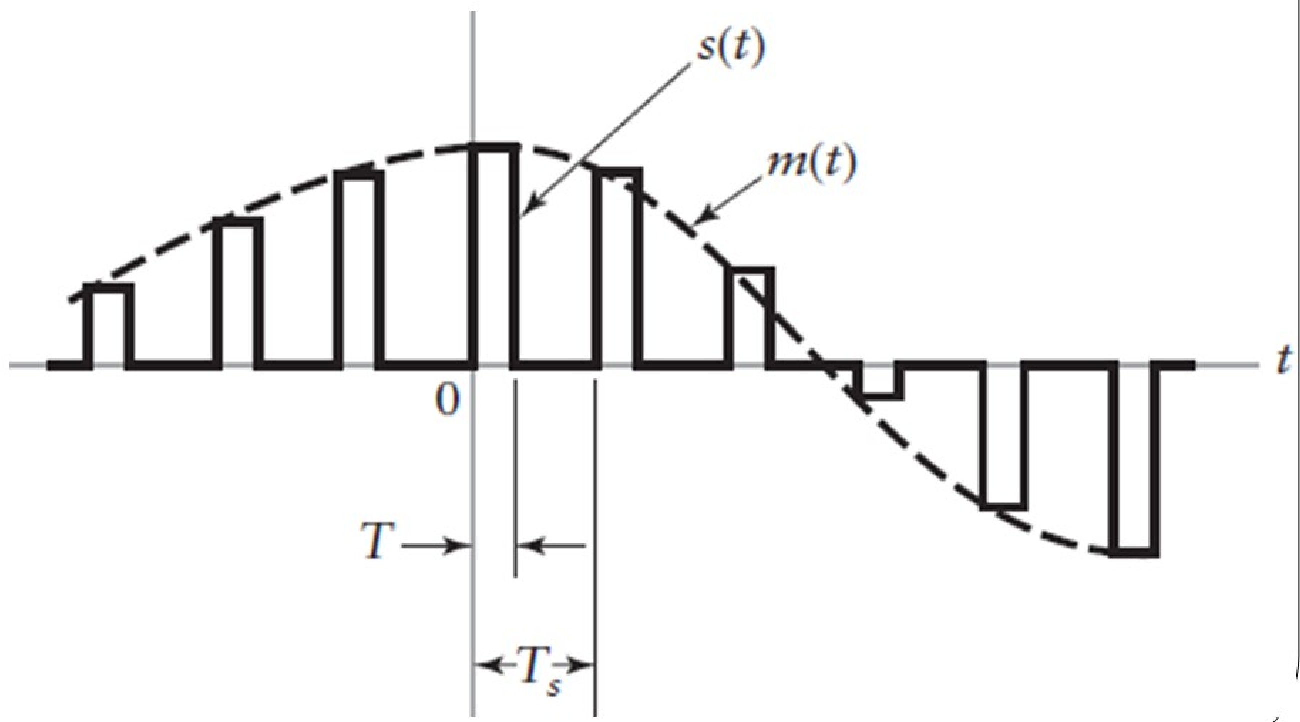


FIGURE 5.5 Flat-top sampling of a message signal.

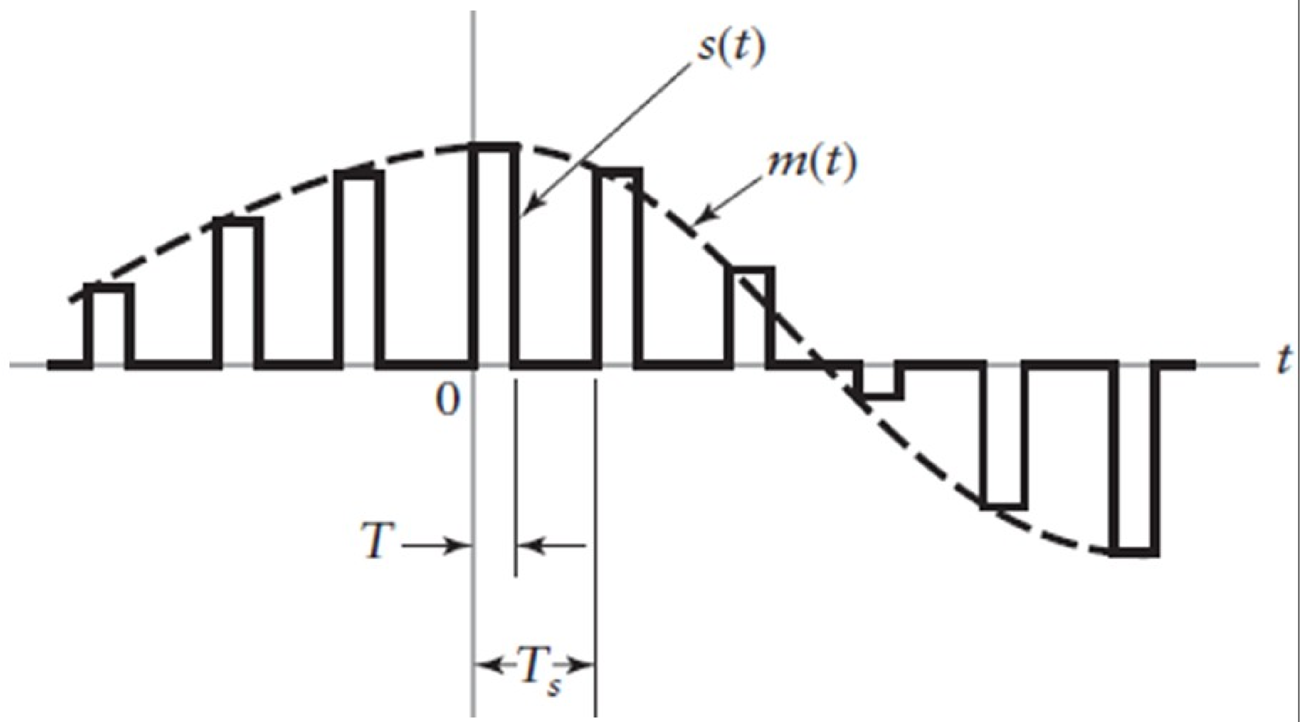
Section 5.2 –Pulse-Amplitude Modulation

There are two operations involved in the generation of the PAM signal:

1. Instantaneous sampling of the message signal m(t) every T_s seconds, where the sampling rate $f_s=1/T_s$ is chosen in accordance with the sampling theorem



Section 5.2 –Pulse-Amplitude Modulation



2. Lengthening the duration of each sample, so that it occupies some finite value T

➢ We refer to these two operations as ***"sample and hold"***

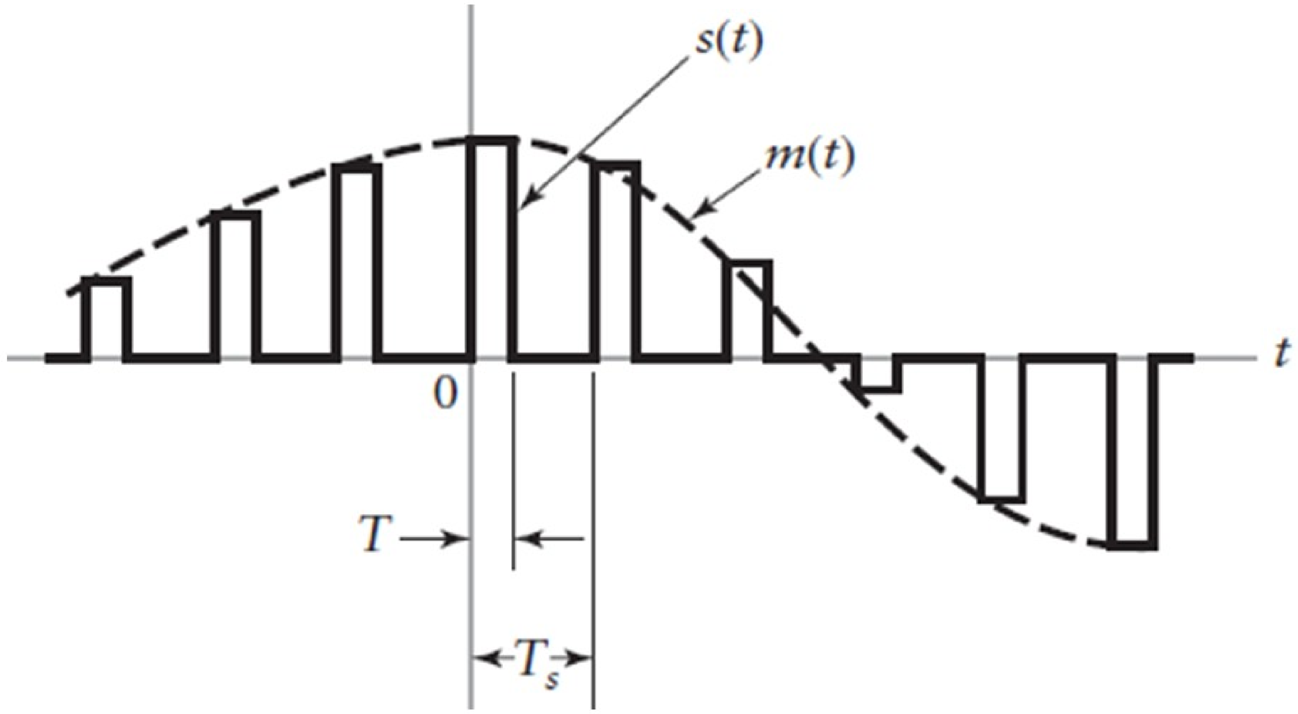
Section 5.2 –Pulse-Amplitude Modulation

- One important reason for intentionally lengthening the duration of each sample is to avoid the use of an excessive channel bandwidth, since ***bandwidth is inversely proportional to pulse duration***, $BW = 1/T$

Section 5.2 – Pulse-Amplitude Modulation

- However, care has to be exercised in how long we make the sample duration T, as the following:

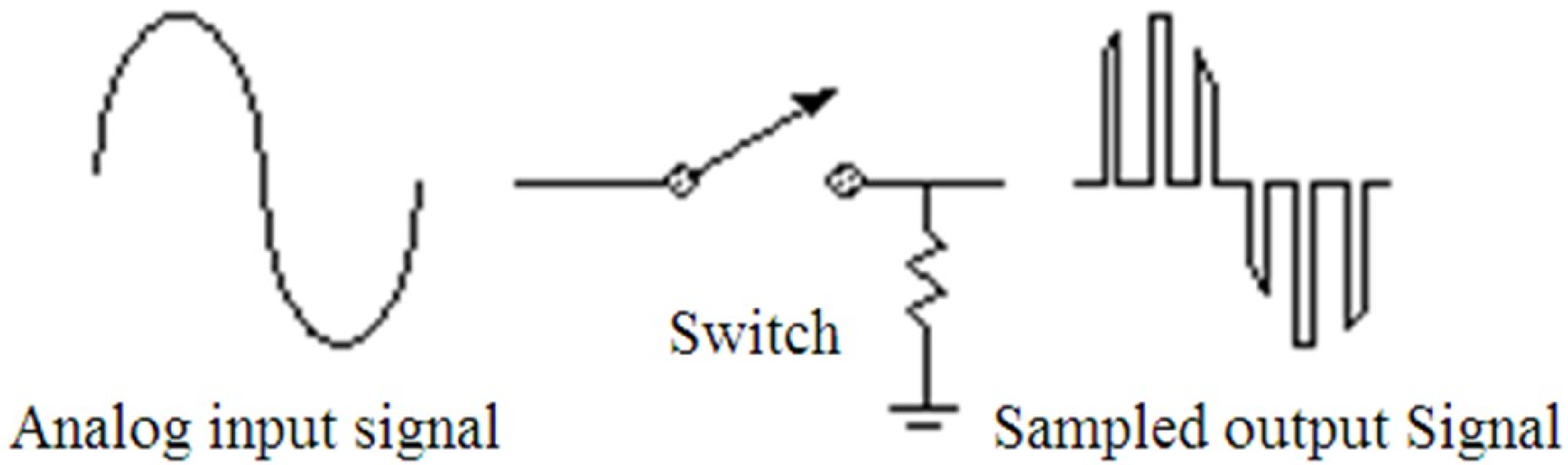
$T/T_s <= 0.1$



Section 5.2 – Pulse-Amplitude Modulation

Natural Sampling

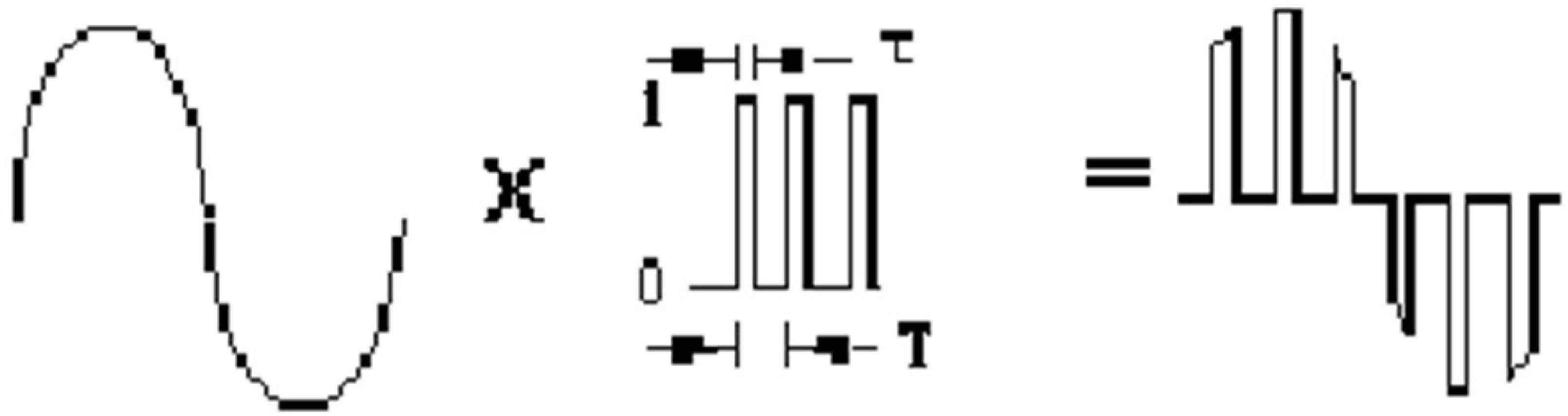
In this type of sampling, the resultant signal follows the natural shape of the input during the sampling interval.



Section 5.2 –Pulse-Amplitude Modulation

Natural Sampling

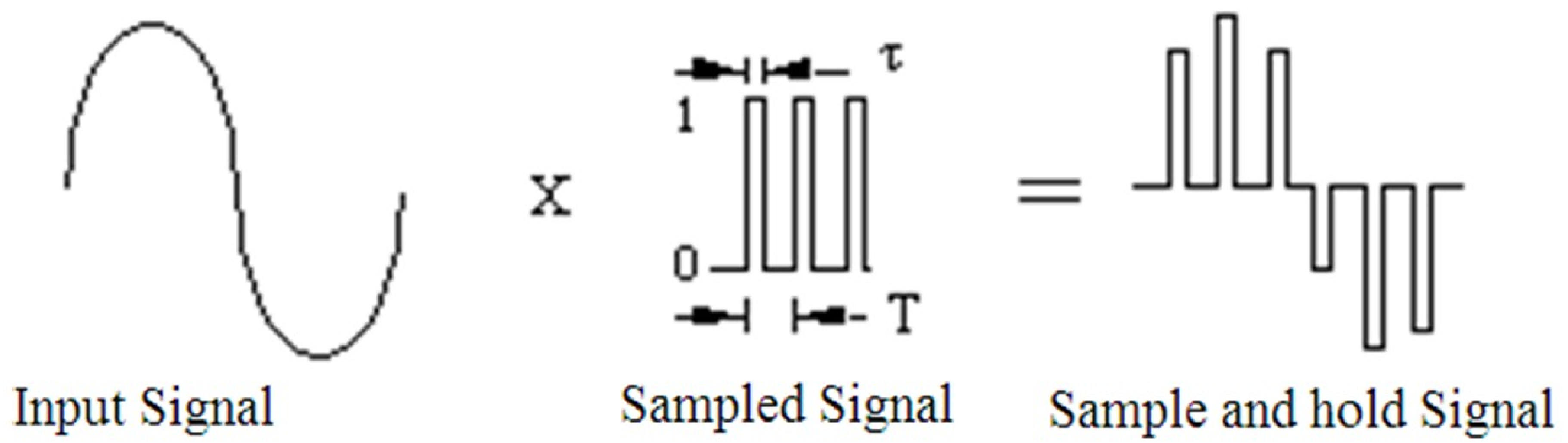
The sampling function can be regarded as a form of ***multiplication***. An output occurs when the input is multiplied by 1, but nothing emerges when it is multiplied by zero



Section 5.2 –Pulse-Amplitude Modulation

Flat Topped Sampling

The sampled signal is ***held constant*** during the conversion process. This alters the time and frequency domain components



Section 5.2 – Pulse-Amplitude Modulation

Sample-and-hold Filter: Analysis - I

Let s(t) denote the sequence of flat-top pulses. Hence, we may express the PAM signal as

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) \qquad (5.8)$$

where

- *Ts* is the sampling period
- *m(nTs)* is the sample value of m(t) obtained at time t= *nTs*
- *h(t)* is a standard rectangular pulse of unit amplitude and duration *T*

$$h(t) = \mathrm{rect}\left(\frac{t - \frac{T}{2}}{T}\right) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0, t = T \\ 0, & \text{otherwise} \end{cases} \qquad (5.9)$$

Section 5.2 – Pulse-Amplitude Modulation

Sample-and-hold Filter: Analysis - II

The instantaneously sampled version of m(t) is given by

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) \qquad (5.10)$$

To get PAM signal s(t), we convolve $m_\delta(t)$ with the pulse h(t)

$$m_\delta(t) \star h(t) = \int_{-\infty}^{\infty} m_\delta(\tau)h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s)h(t-\tau)\,d\tau$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t-\tau)\,d\tau \qquad (5.11)$$

Section 5.2 –Pulse-Amplitude Modulation

Sample-and-hold Filter: Analysis - III

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s) h(t - \tau) \, d\tau \qquad (5.11)$$

Using the sifting property of the delta function—namely

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s) h(t - \tau) \, d\tau = h(t - nT_s)$$

we find that Eq. (5.11) reduces to

$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s) h(t - nT_s) \qquad (5.12)$$

$$s(t) = m_\delta(t) \star h(t) \qquad (5.13)$$

Section 5.2 – Pulse-Amplitude Modulation

Sample-and-hold Filter: Analysis - IV

$$s(t) = m_\delta(t) \star h(t) \tag{5.13}$$

Taking the Fourier transform of both sides, we get

$$S(f) = M_\delta(f)H(f) \tag{5.14}$$

From Eq. (5.2)

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \tag{5.15}$$

So S(f) is given by

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f) \tag{5.16}$$

Section 5.2 –Pulse-Amplitude Modulation

Aperture Effect And Its Equalization - I

Recovering the original message signal m(t) from s(t):

FIGURE 5.7 Recovering the message signal $m(t)$ from the PAM signal $s(t)$.

Section 5.2 –Pulse-Amplitude Modulation

Aperture Effect And Its Equalization - II

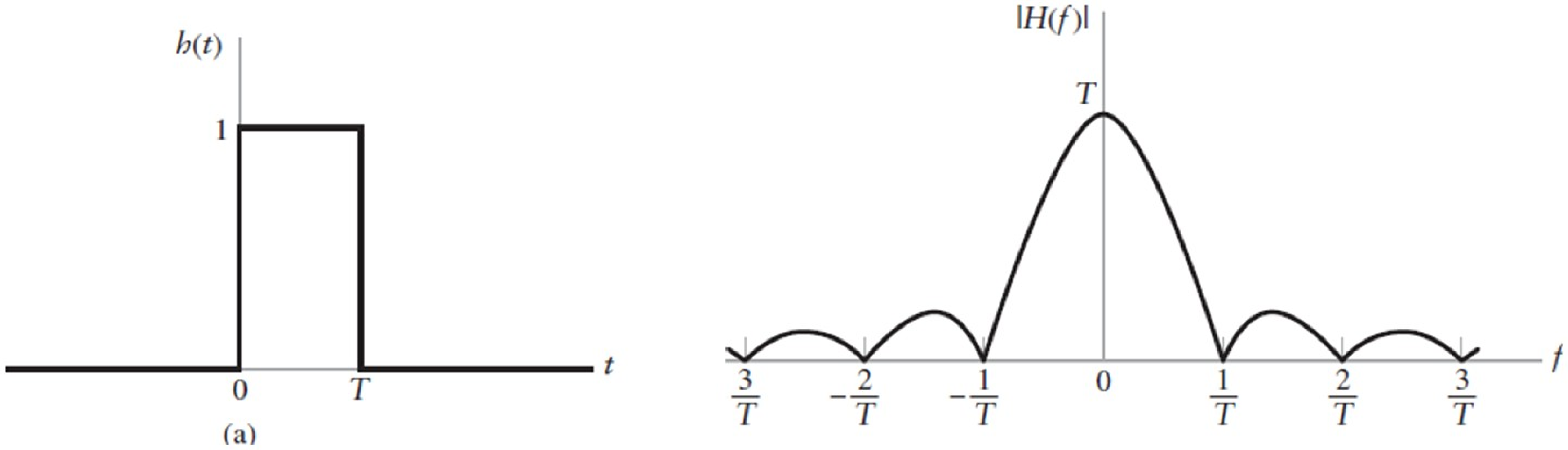


FIGURE 5.6 (*a*) Rectangular pulse $h(t)$. (*b*) Spectrum $H(f)$

By using flat-top samples to generate a PAM signal, we have introduced amplitude distortion as well as a delay of T/2

Section 5.2 –Pulse-Amplitude Modulation

Aperture Effect And Its Equalization - III

This distortion may be corrected by connecting an ***equalizer*** in cascade with the ***low-pass reconstruction*** filter

FIGURE 5.7 Recovering the message signal $m(t)$ from the PAM signal $s(t)$.

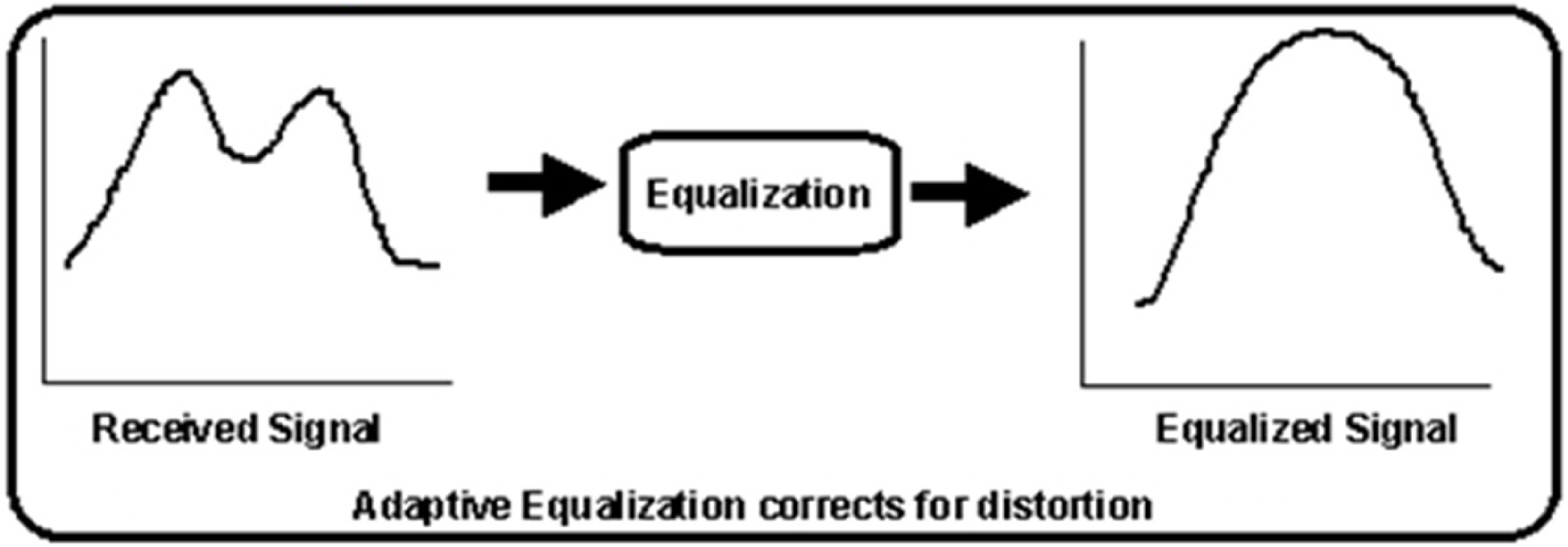
- ***Equalization,*** the process of adjusting the strength of certain frequencies within a signal

Section 5.2 –Pulse-Amplitude Modulation

Aperture Effect And Its Equalization - IV

The equalizer has the effect of decreasing the in-band loss of the reconstruction filter as the frequency increases in such a manner as to compensate for the aperture effect

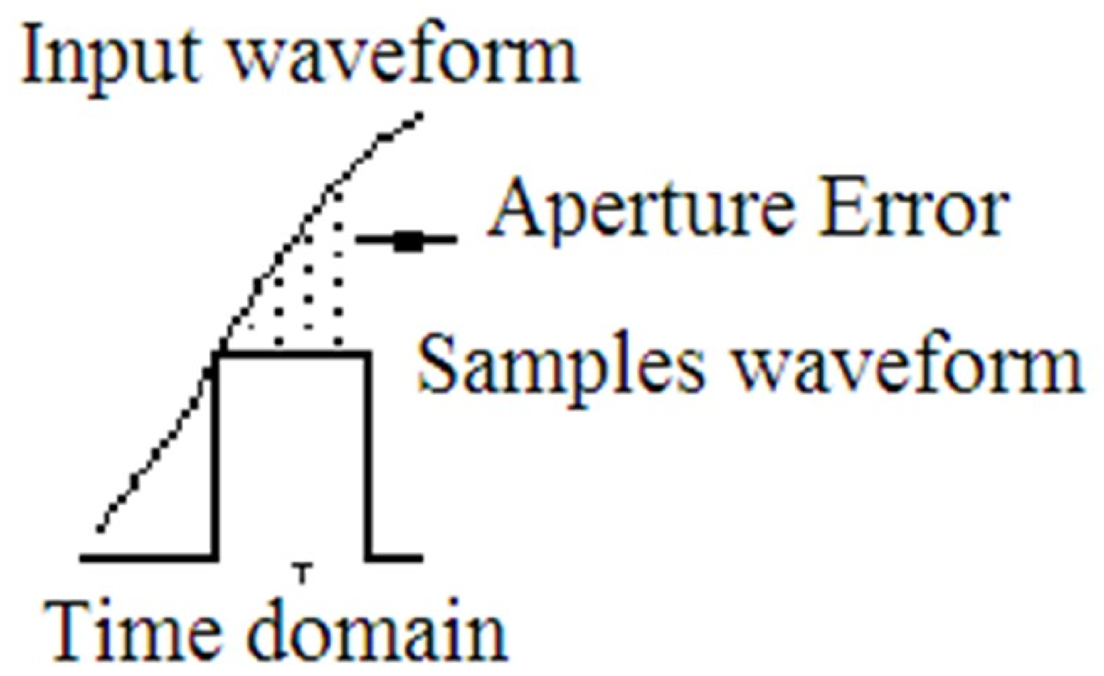
*external example



Section 5.2 –Pulse-Amplitude Modulation

Aperture Error (effect)

Aperture error is the difference between the actual value of the input signal, and the flat-topped sample value



Section 5.2 –Pulse-Amplitude Modulation

Aperture Effect And Its Equalization -V

For a duty cycle $T/T_s \leq 0.1$, the amplitude distortion is less than 0.5 percent, in which case the need for equalization may be omitted altogether

Section 5.3 – Pulse-Position Modulation

Section 5.3 –Pulse-Position Modulation

In ***Pulse-Duration Modulation (PDM),*** the samples of the message signal are used to vary the duration of the individual pulses. We refer to it also by:

- ***Pulse-Width Modulation or Pulse-Length Modulation***

- The modulating signal m(t) may vary the time of occurrence of the leading edge, the trailing edge, or both edges of the pulse

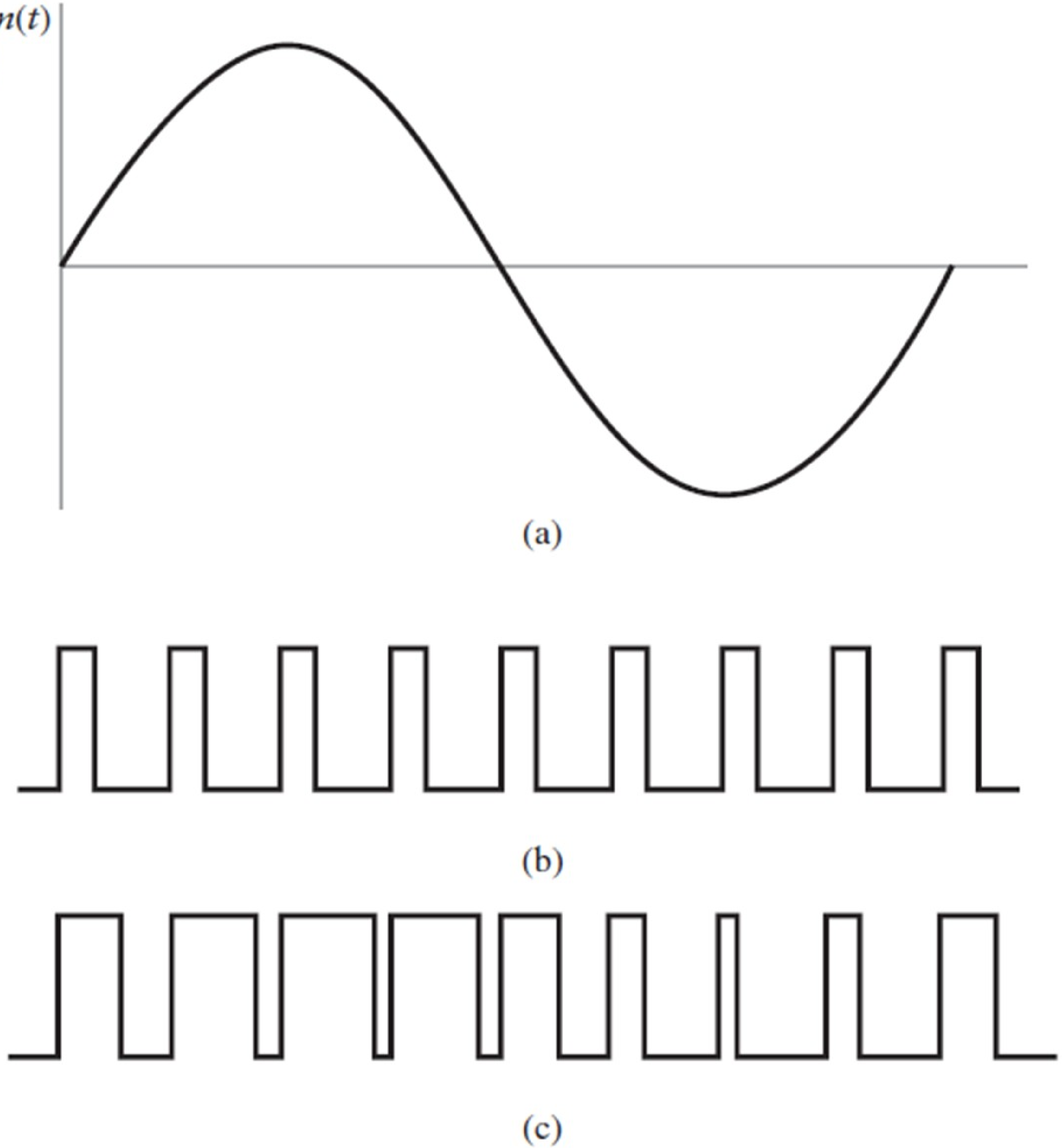
Section 5.3 –Pulse-Position Modulation

Pulse-Duration Modulation

(a) Modulating wave.

(b) Pulse carrier

(c) PDM wave



Dr. Ahmed Masri

Section 5.3 – Pulse-Position Modulation

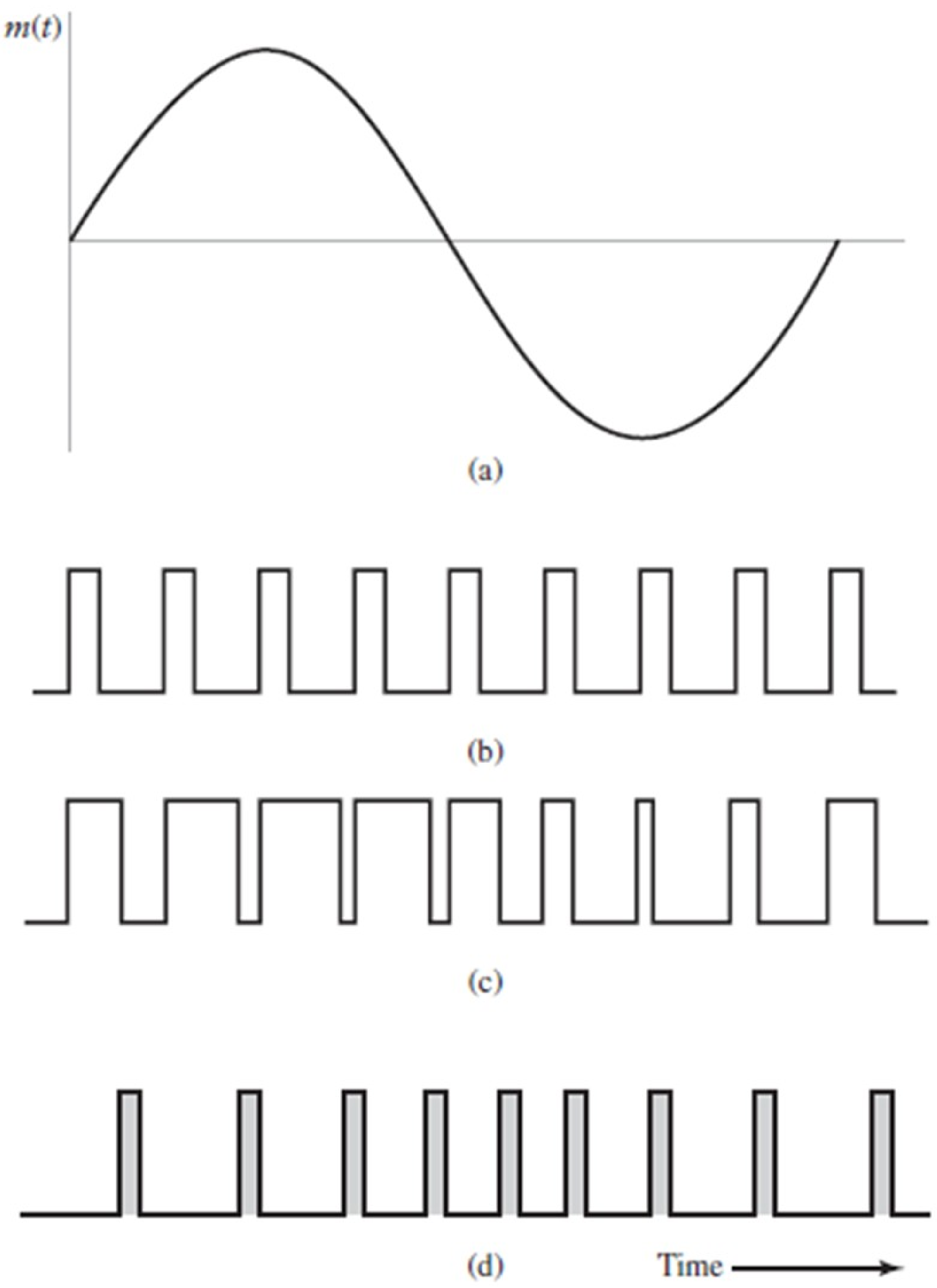
Pulse-Duration Modulation

- PDM is wasteful of power, in that long pulses expend considerable power during the pulse while bearing no additional information
- Power saving through considering only time transitions
- This will lead to ***Pulse-Position Modulation (PPM)***

Section 5.3 – Pulse-Position Modulation

Pulse-Position Modulation

- In PPM, the position of a pulse relative to its ***unmodulated time*** of occurrence is varied in accordance with the message signal



Section 5.3 – Pulse-Position Modulation

Pulse-Position Modulation

- The PPM signal

$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s)) \qquad (5.18)$$

where $\boldsymbol{k_p}$ is the sensitivity factor of the pulse-position modulator (in seconds per volt)

- The different pulses constituting the PPM signal s(t) must be strictly nonoverlapping

$$g(t) = 0, \qquad |t| > (T_s/2) - k_p|m(t)|_{\max} \qquad (5.19)$$

$$k_p|m(t)|_{\max} < (T_s/2) \qquad (5.20)$$