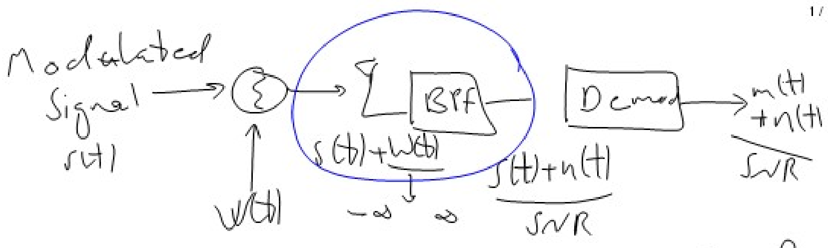


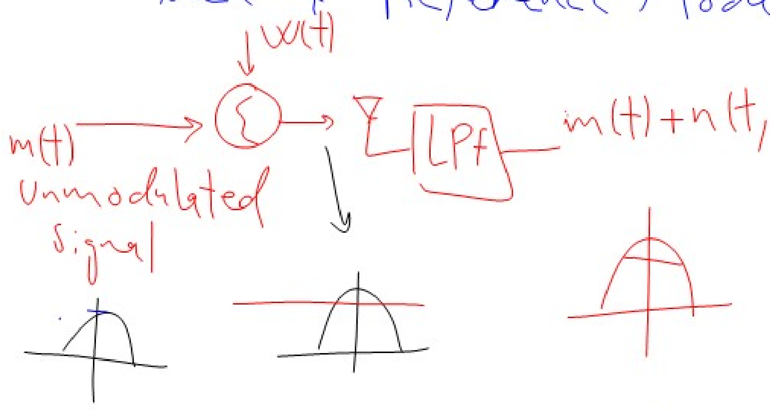
s(t)+n(t): SNR measured at the input of demodulator "pre-detection SNR"

→ it measures the quality of the transmission link and receiver frontend

(2) m(t)+n(t): SNR measured at the output of the Receiver
"post-detection SNR"

→ it measures the quality of the demodulation process and the quality of the info received.

To Compare different modulation schemes, we introduce the idea of "Reference Model"



⇒ This model is equivalent to transmitting at baseband.

We made two assumptions:

(1) The msg power is the same as the modulated s(t) signal power

(2) The baseband LPF passes the msg signal and rejects out of band noise

We may define "Reference SNR"

$$SNR_{ref} = \frac{\text{avg power of the modulated signal}}{\text{avg power of the noise measured within BW of } m(t)}$$

$$= \frac{\overline{|s(t)|^2}}{\overline{|n(t)|^2}}$$

We may define "Figure of merit" to compare diff mod-dem schemes

$$F = \frac{\text{Post-detection SNR}}{\text{Reference SNR}}$$

the higher figure of merit the better the system is.

Communications and Signals Processing

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2013/2014

Chapter 9- Outlines

9.1 Noise in Communication Systems

9.2 Signal-to-Noise Ratios

9.3 Band-Pass Receiver Structures

9.4 Noise in Linear Receivers Using Coherent Detection

9.5 Noise in AM Receivers Using Envelope Detection

9.6 Noise in SSB Receivers

9.7 Detection of Frequency Modulation (FM)

9.8 FM Pre-emphasis and De-emphasis

9.9 Summary and Discussion

9.1: Noise in Communication Systems

Written Notes delivered to the student.

Section 9.2: Signal-To-Noise Ratios

9.2: Signal-To-Noise Ratios

Part of it is written notes delivered to the student.

9.2: Signal-To-Noise Ratios

In order to compare different ***analog modulation–demodulation schemes***, we introduce the idea of a ***reference transmission model*** as depicted in Fig.

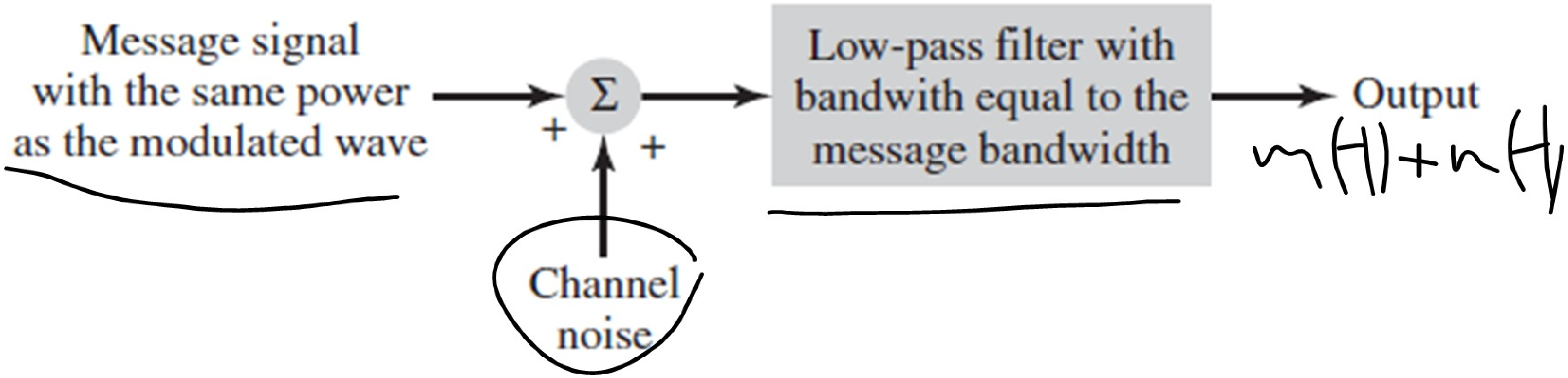

FIGURE 9.4 Reference transmission model for analog communications.

- This reference model is equivalent to transmitting the message at ***baseband***

9.2: Signal-To-Noise Ratios

In this model, two assumptions are made:

1. The message power is the same as the modulated signal power of the modulation scheme under study.
2. The baseband low-pass filter passes the message signal and rejects out-of-band noise.

Accordingly, we may define the reference signal-to-noise ratio, $\boldsymbol{SNR_{ref}}$ as

$$\mathrm{SNR_{ref}} = \frac{\text{average power of the modulated message signal}}{\text{average power of noise measured in the message bandwidth}} \quad (9.11)$$

9.2: Signal-To-Noise Ratios

- The reference signal-to-noise ratio of Eq. (9.11) may be used to compare different modulation–demodulation schemes by using it ***to normalize*** the post-detection signal-to-noise ratios
- That is, we may define a ***figure of merit*** for a particular modulation–demodulation scheme as follows:

$$\text{Figure of merit} = \frac{\text{post}-\text{detection SNR}}{\text{reference SNR}}$$

9.2: Signal-To-Noise Ratios

$$\text{Figure of merit} = \frac{\text{post-detection SNR}}{\text{reference SNR}}$$

- Clearly, the ***higher*** the value that the figure of merit has, the ***better*** the noise performance of the receiver will be.

9.2: Signal-To-Noise Ratios

To summarize our consideration of signal-to-noise ratios:

- The ***pre-detection*** SNR is measured before the signal is demodulated.
- The ***post-detection*** SNR is measured after the signal is demodulated.
- The ***reference SNR*** is defined on the basis of a baseband transmission model.
- The ***figure of merit*** is a dimensionless metric for comparing different analog modulation–demodulation schemes and is defined as the ratio of the post-detection and reference SNRs.

Section 9.3: Band-Pass Receiver Structures

9.3: Band-Pass Receiver Structures

- The transmitter includes a modulator that produces an output at a standard ***intermediate frequency (IF)*** and a local mixer-translates (up-converts) this output to a "channel" or ***radio frequency (RF).***

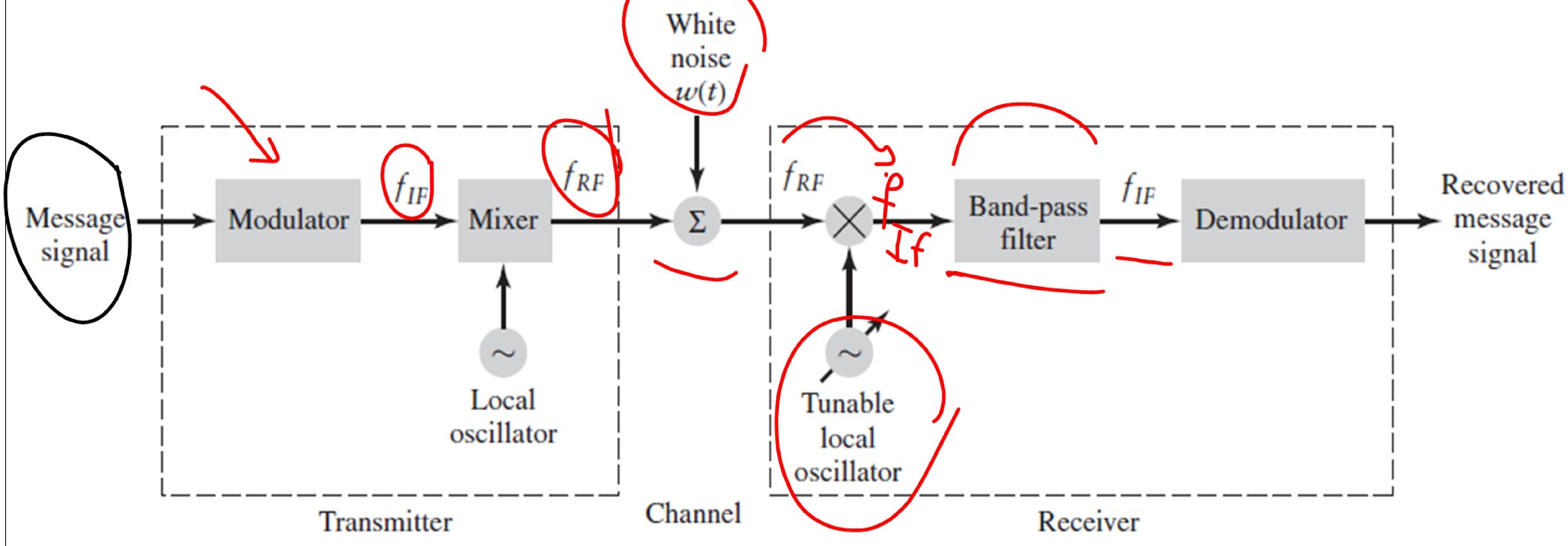

FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

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9.3: Band-Pass Receiver Structures

- The right-hand side of Fig. 9.5 shows an example of a ***super-heterodyne receiver*** that was discussed

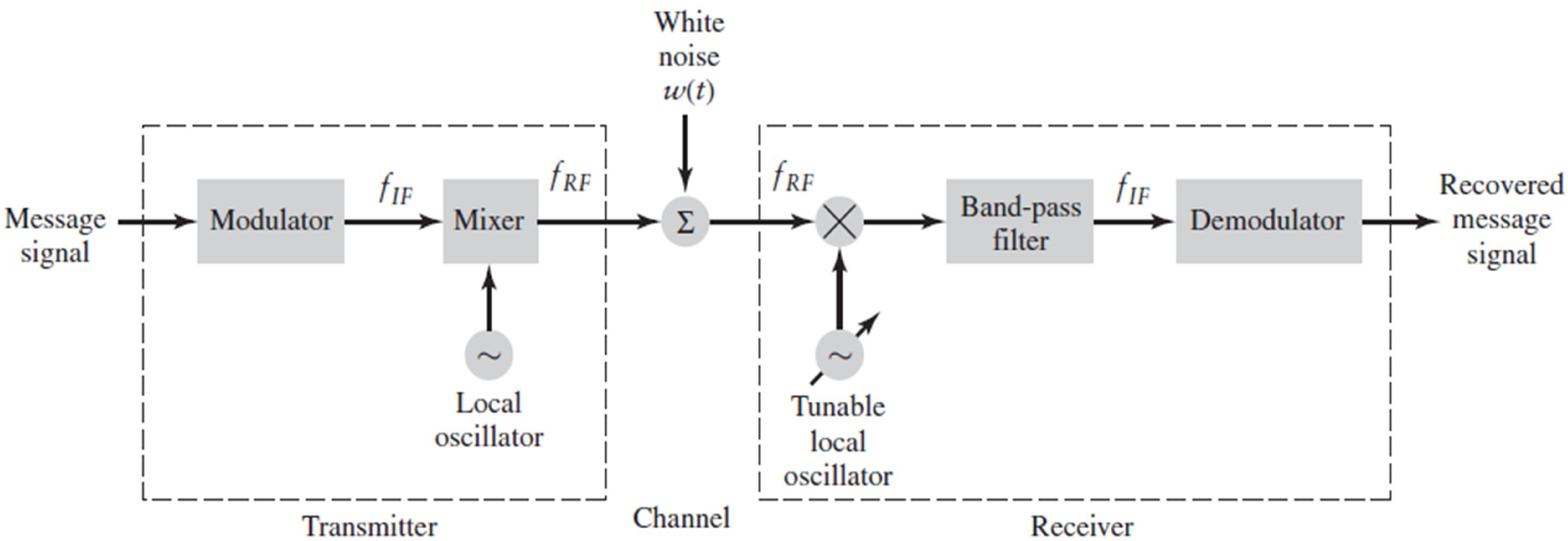

FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

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9.3: Band-Pass Receiver Structures

- At the receiver, a tunable local oscillator frequency-translates (***down-converts***) this ***channel frequency (RF)*** to a standard ***intermediate frequency (IF)*** for demodulation

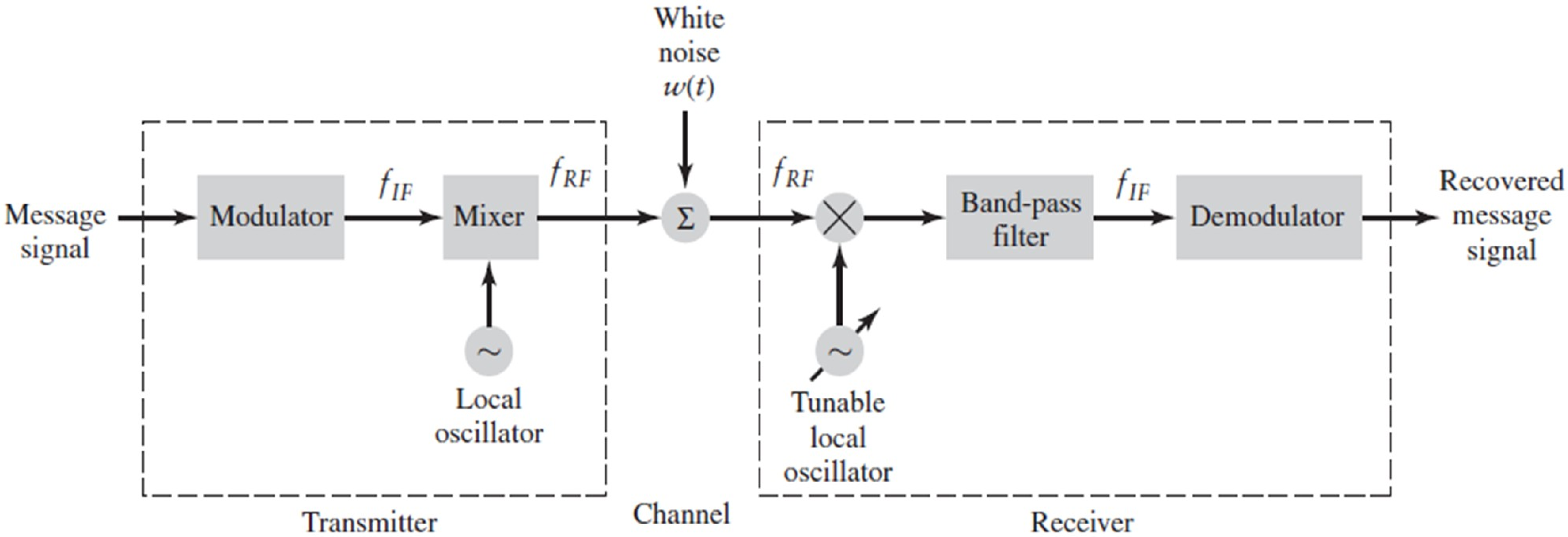


FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

9.3: Band-Pass Receiver Structures

- Common *examples* are AM radio transmissions, where the RF channels' frequencies lie in the range between 510 and 1600 kHz, and a common IF is 455 kHz; another example is FM radio, where the RF channels are in the range from 88 to 108 MHz and the IF is typically 10.7 MHz.

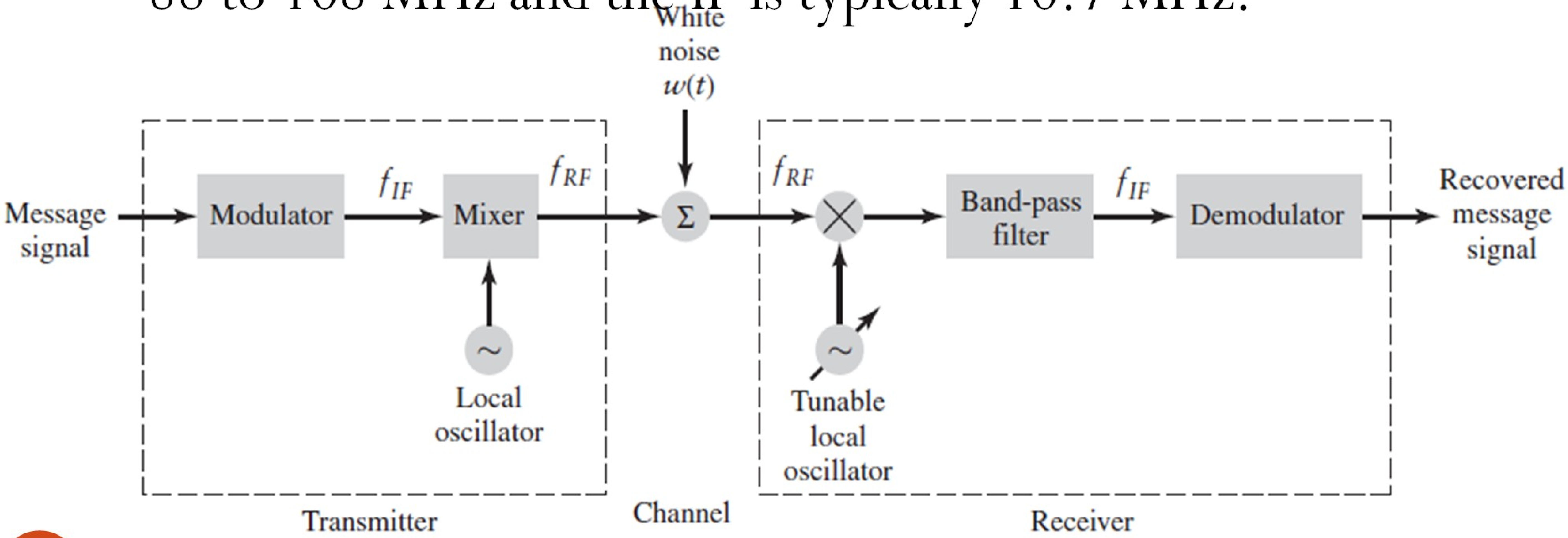


FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

9.3: Band-Pass Receiver Structures

- Band-pass signals using the in-phase and quadrature representation with

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

where is $S_I(t)$ the in-phase component of and $S_Q(t)$ is its quadrature component

9.3: Band-Pass Receiver Structures

- Most receivers immediately limit the white noise power by processing the received signal with a band-pass filter
- Typically, there is a band-pass filter before and after the local oscillator

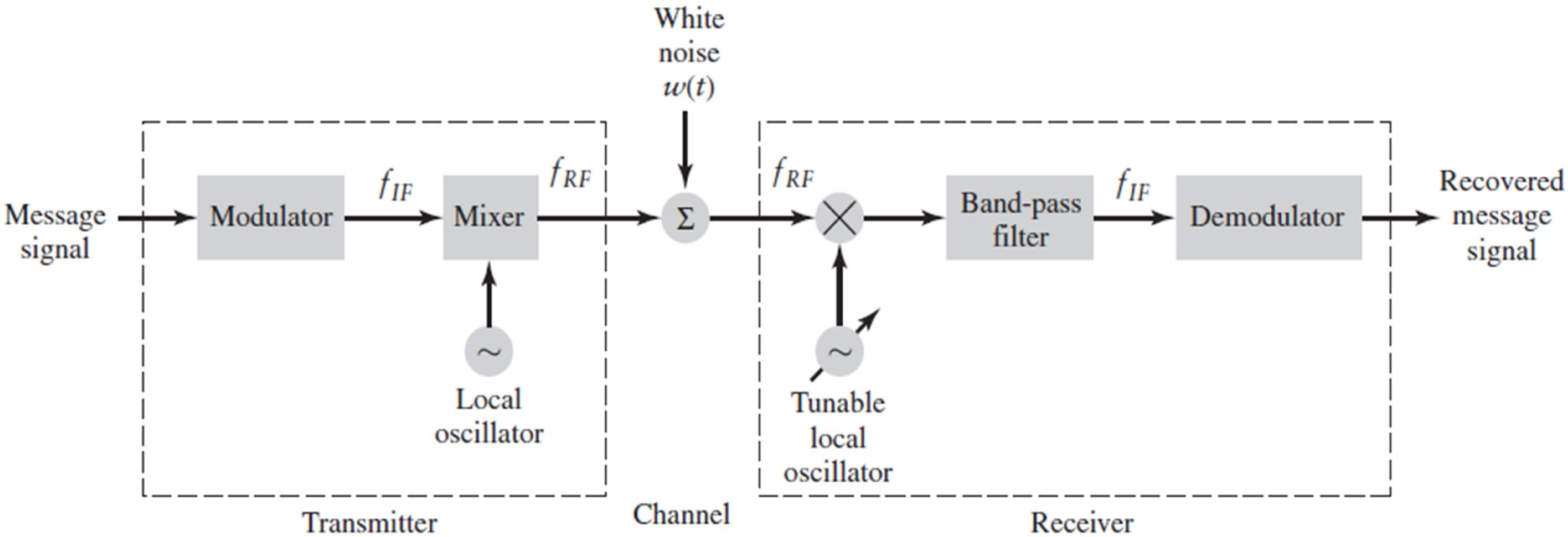
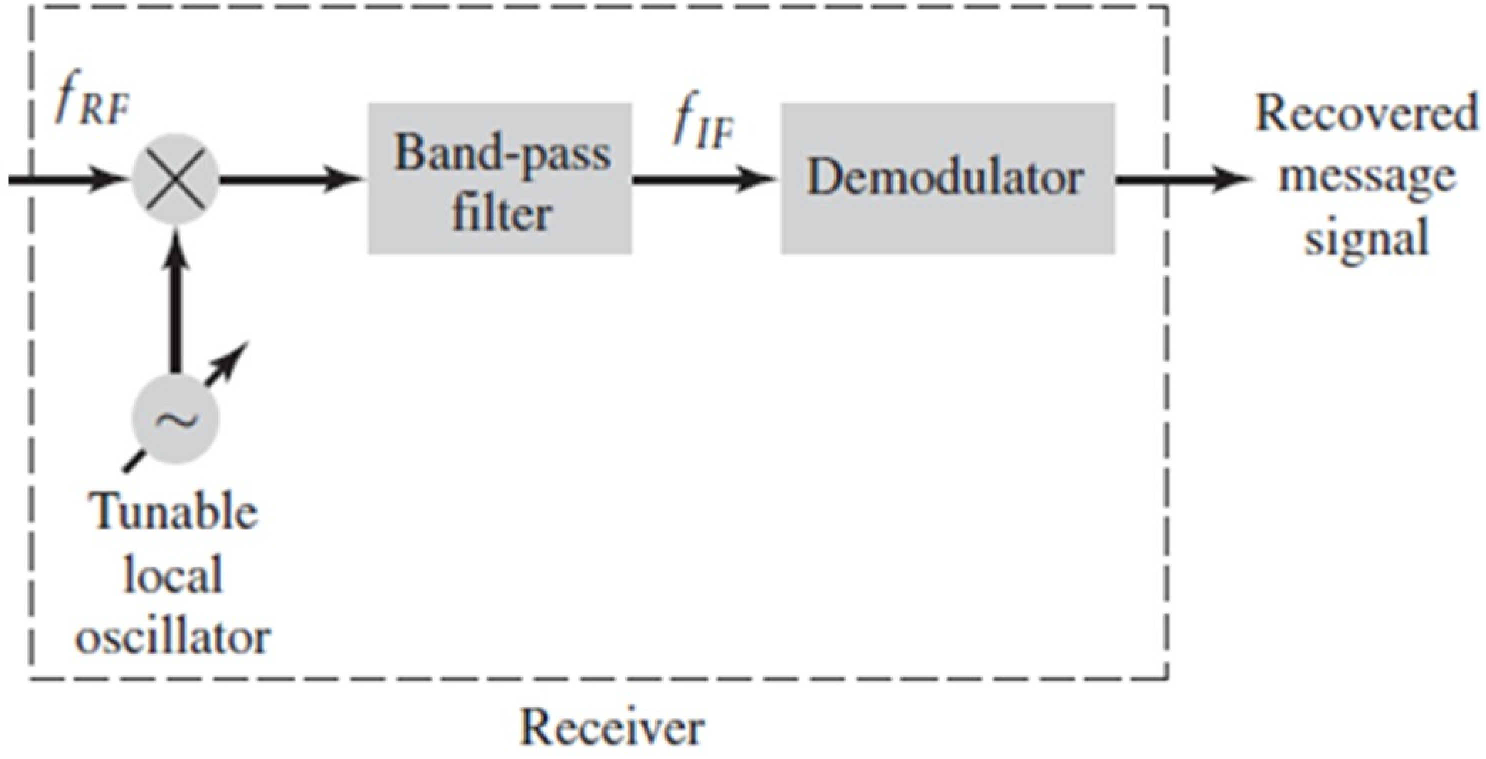


FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

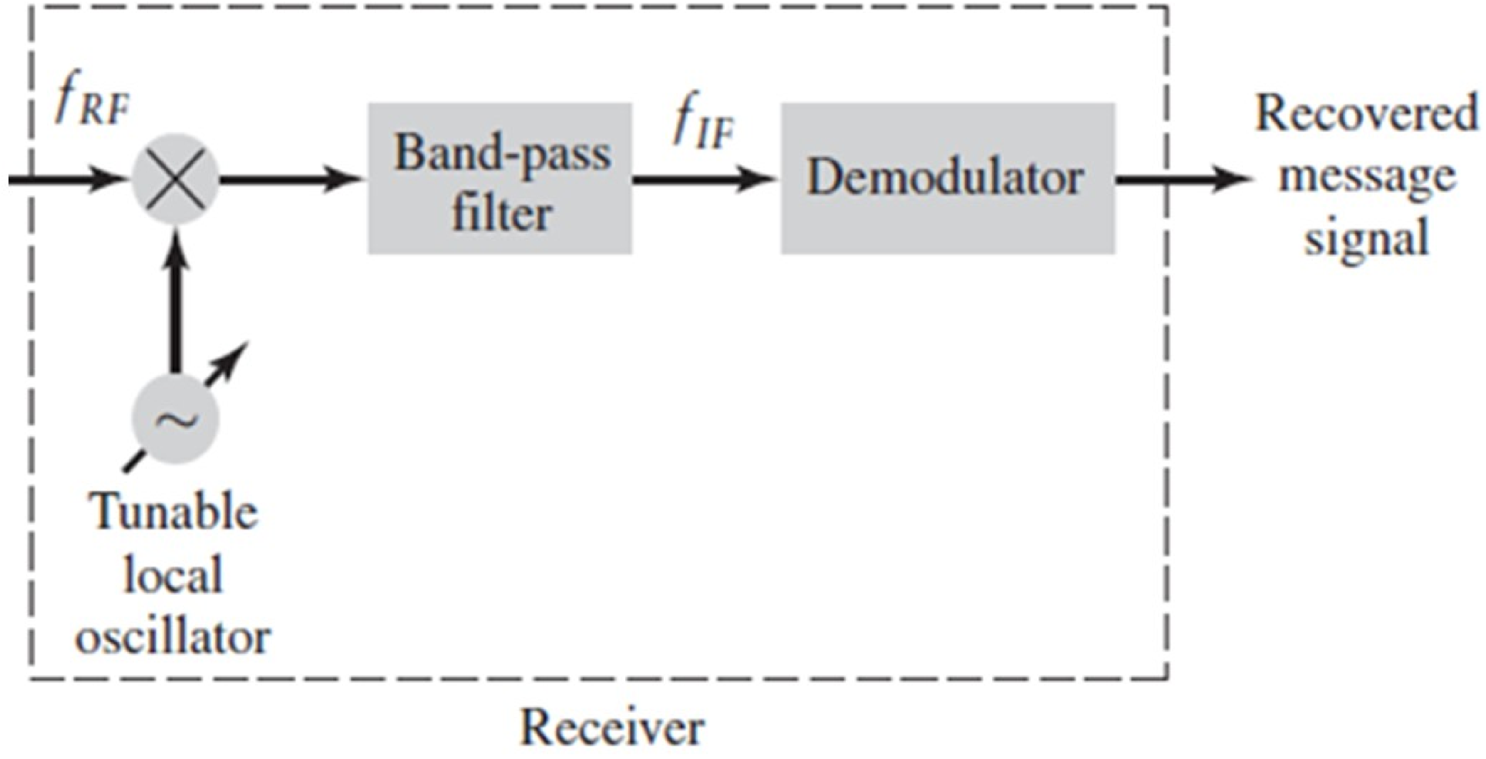
9.3: Band-Pass Receiver Structures

- The *filter before* the local oscillator is centered at a higher RF frequency and is usually much wider, wide enough to encompass all RF channels that the receiver is intended to handle
- **For example**, with an FM receiver the band-pass filter before the local oscillator would pass the frequencies from 88 to 108 MHz



9.3: Band-Pass Receiver Structures

- The *band-pass filter after* the oscillator passes the signal of a single RF channel relatively undistorted but limits the noise to those components within the passband of the filter.
- With the same FM receiver, the band-pass filter after the local oscillator would be approximately 200 kHz wide; ***it is the effects of this narrower filter that are of most interest to us***



Section 9.4: Noise in Linear Receivers Using Coherent Detection

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme:

- In the case of double side band suppressed-carrier (DSB-SC) modulation, the modulated signal is represented as

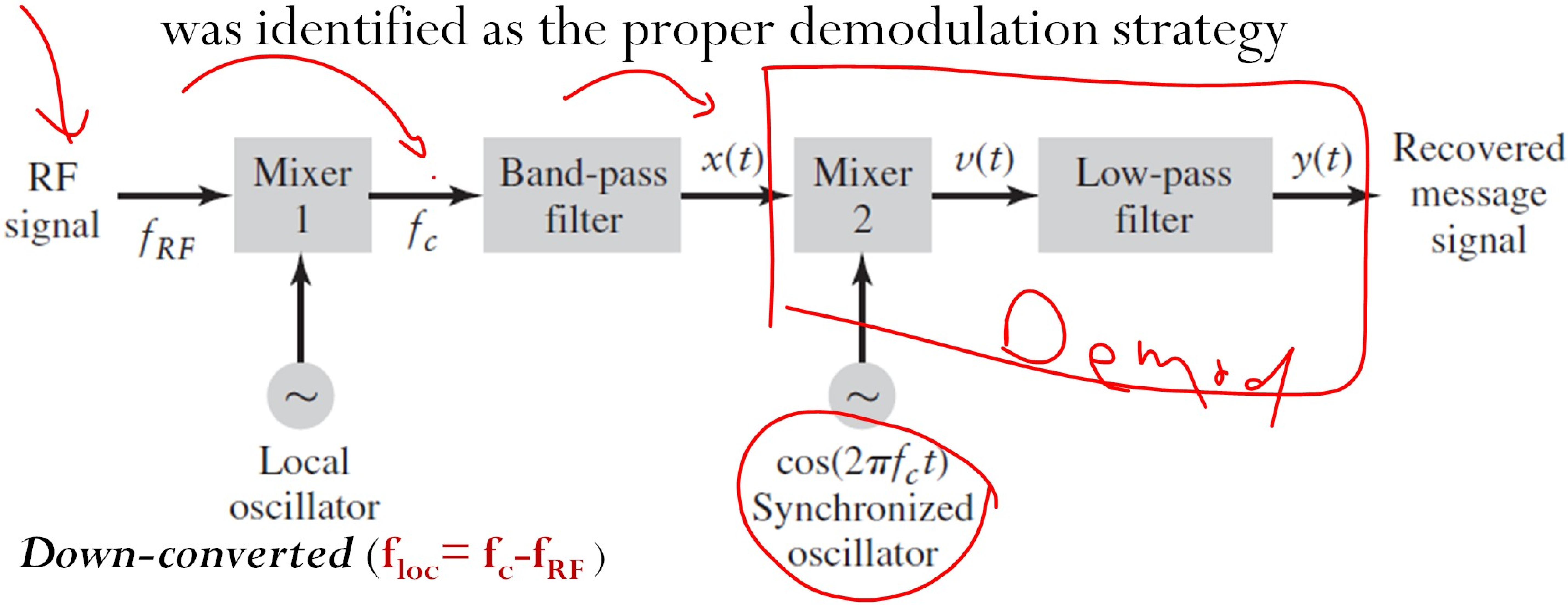
$$s(t) = A_c m(t) \cos(2\pi f_c t + \theta)$$

- Where $\mathbf{f_c}$ is the carrier frequency, and $\mathbf{m(t)}$ is the message signal; the carrier phase θ is a random variable, but not varying during the course of transmission

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme:

- For suppressed-carrier signals, linear coherent detection was identified as the proper demodulation strategy



Down-converted ($f_{loc} = f_c - f_{RF}$)

Figure 9.6 A linear DSB-SC receiver using coherent demodulation.

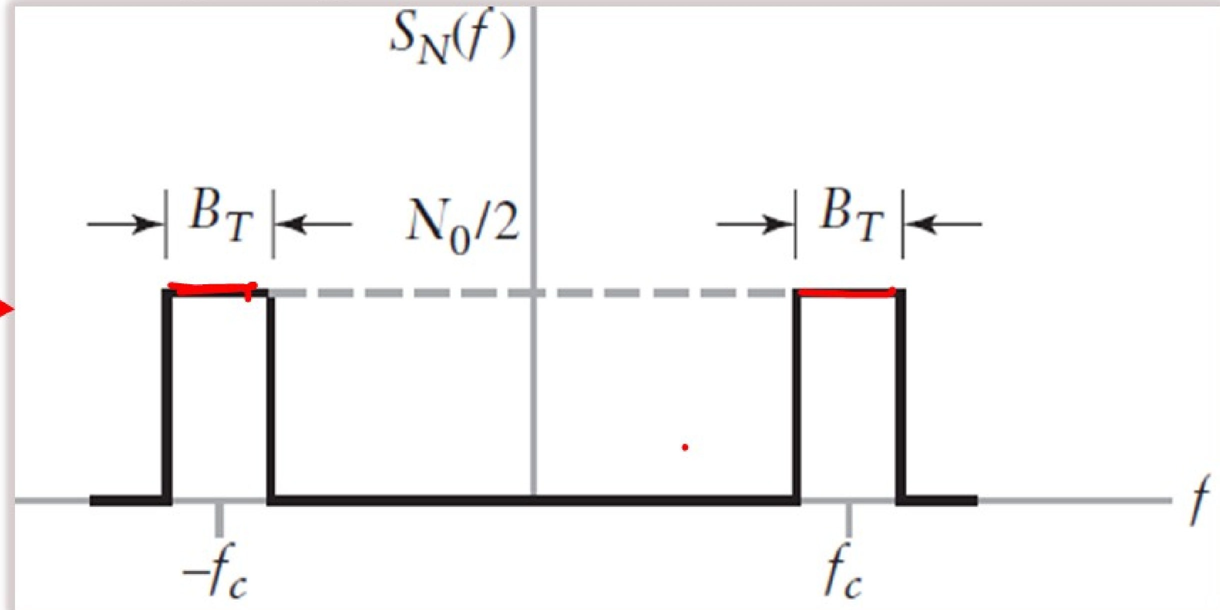
9.4: Noise in Linear Receivers Using Coherent Detection

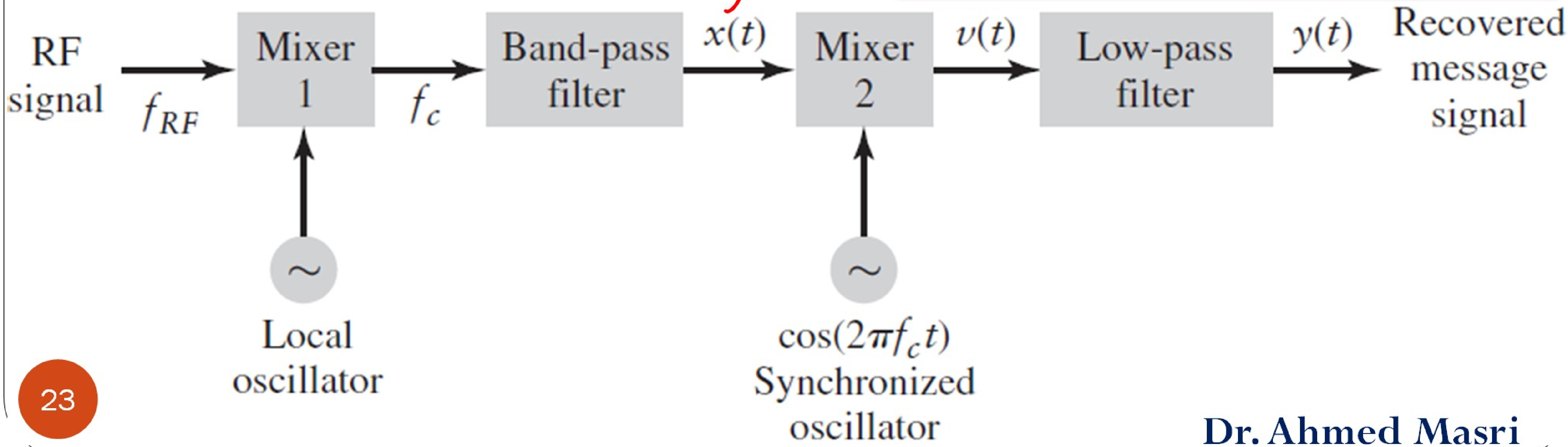
Noise in DSB-SC modulation scheme:

- After band-pass filtering, the resulting signal is

$$x(t) = s(t) + n(t)$$

s(t)+w(t)





9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Pre-detection SNR

- The average power of the signal s(t):
  - Since the carrier and modulating signal are independent, this can be broken down into two components as follows:

$$\mathrm{E}[s^2(t)] = \mathrm{E}[(A_c \cos(2\pi f_c t + \theta))^2]\mathrm{E}[m^2(t)]$$

- If we let

$$P = \mathrm{E}[m^2(t)]$$

be the average signal (message) power and using the result of Example "SNR of sinusoidal"

$$\mathrm{E}[s^2(t)] = \frac{A_c^2 P}{2}$$

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Pre-detection SNR

- If the band-pass filter has a noise bandwidth $\mathbf{B_T}$ then the noise power passed by this filter is $\mathbf{N_0 B_T}$ by definition
- Consequently, the signal-to-noise ratio of the signal is

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2 N_0 B_T}$$

- This is the ***pre-detection signal-to-noise ratio*** of the DSB-SC system ***because*** it is measured at a point in the system ***before*** the message signal **m(t)** is demodulated

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Post-detection SNR

- The post-detection signal-to-noise ratio is the ratio of the message signal power to the noise power *after* demodulation/detection
- The post-detection SNR depends on both the modulation and demodulation techniques

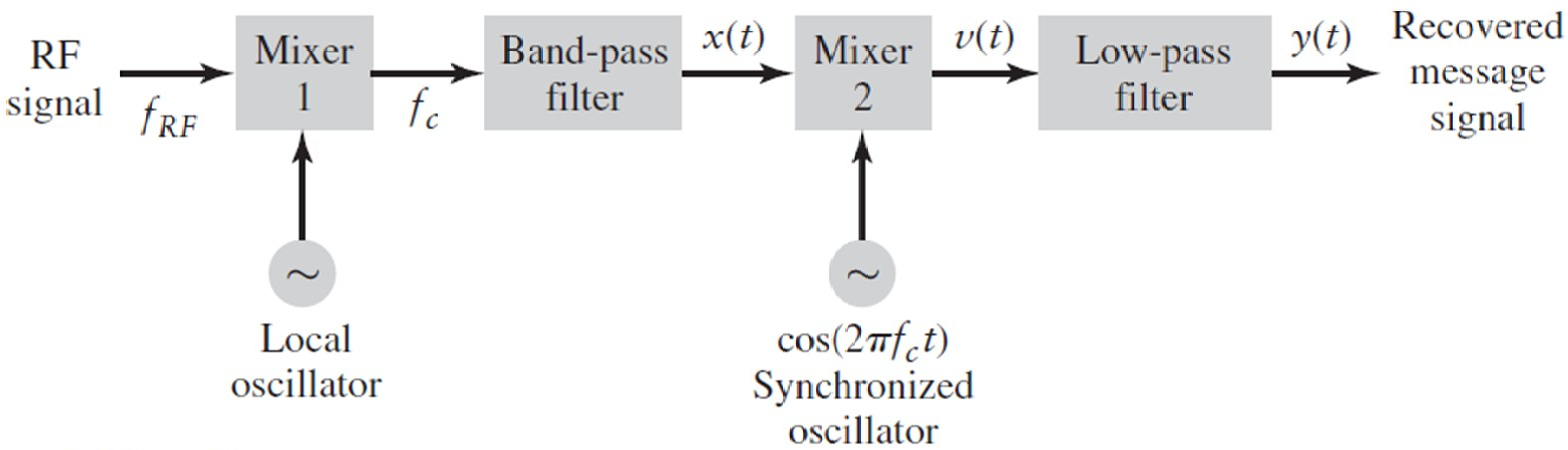
9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Post-detection SNR

- Using the narrowband representation of the band-pass noise, the signal at the input to the coherent detector

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

- where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature components of $n(t)$ with respect to the carrier



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9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Post-detection SNR

- The output of mixer 2

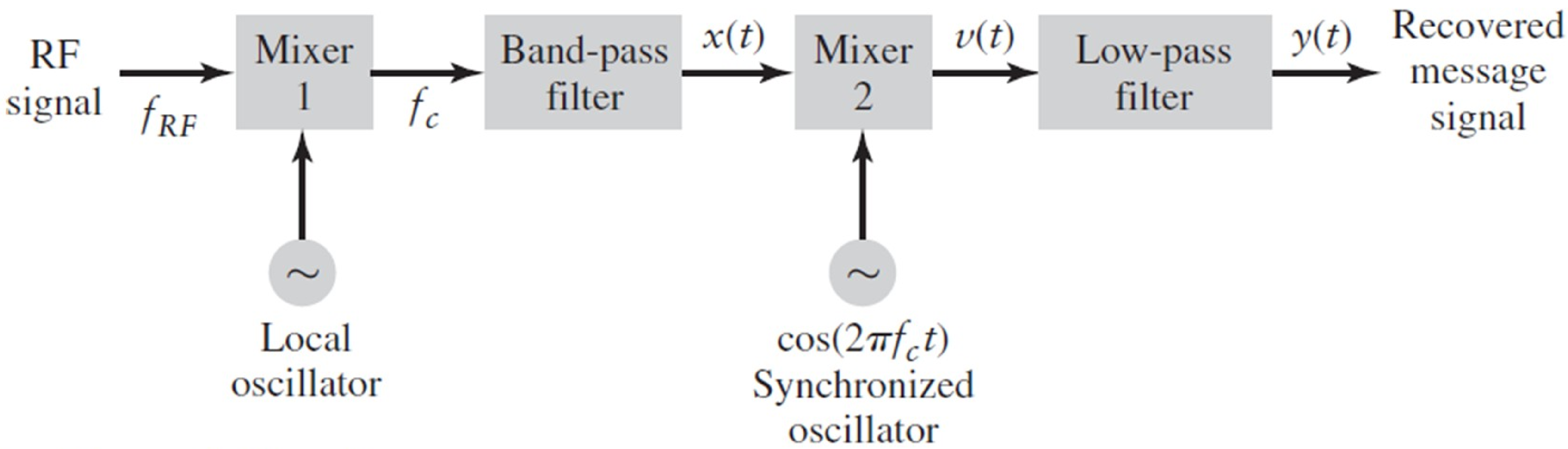
$$\cos A \cos A = \frac{1 + \cos 2A}{2}$$

$$\sin A \cos A = \frac{\sin 2A}{2}$$

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= \tfrac{1}{2}(A_c m(t) + n_I(t))$$

$$+\tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2}n_Q(t)\sin(4\pi f_c t)$$



9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Post-detection SNR

$$= \frac{1}{2}(A_c m(t) + n_I(t))$$
$$+ \frac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t)$$

- The first part represents the ***baseband signal*** and ***in-phase component of the noise***, while the second part represents ***quadrature component*** of its noise centered at the much higher frequency of $2\mathbf{f}_c$
- These high-frequency components are removed with a low-pass filter

$$y(t) = \frac{1}{2}(A_c m(t) + n_I(t))$$

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: **Post-detection SNR**

Two observations can be made:

$$y(t) = \frac{1}{2}(A_c m(t) + n_I(t))$$

- The message signal and the in-phase component of the filtered noise appear additively in the output.
- The quadrature component of the noise is completely rejected by the demodulator.

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: **Post-detection SNR**

Now we may compute the output or post-detection signal to noise ratio by noting the following:

1) The message component $\frac{1}{2}A_c m(t)$ is so analogous to the computation of the predetection signal power, the post-detection signal power is $\frac{1}{4}A_c^2 P$ where P is the average message power as defined $P = \mathrm{E}[m^2(t)]$

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: **Post-detection SNR**

2) The noise component $\frac{1}{2}n_I(t)$ is after low-pass filtering. The in-phase component has a noise spectral density of N_0 over the bandwidth from $-B_T/2$ to $B_T/2$

- If the low-pass filter has a noise bandwidth W, corresponding to the message bandwidth, which is less than or equal to $B_T/2$ then the output noise power is

$$\mathrm{E}[n_I^2(t)] = \int_{-W}^{W} N_0 \, df = 2N_0 W$$

- Thus the power in $n_I(t)$ is $2N_0W$.

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Post-detection SNR

- Combining these observations, we obtain the post-detection SNR of

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} = \frac{\frac{1}{4}(A_c^2)P}{\frac{1}{4}(2N_0W)} = \frac{A_c^2 P}{2N_0 W}$$

- Consequently, if $W \approx B_T/2$, the post-detection SNR is *twice* the pre-detection SNR.
- *This is due to the fact that the quadrature component of the noise has been discarded by the synchronous demodulator.*

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme: Figure Of Merit

- It should be clear that for the reference transmission model defined in Section 9.2, the average noise power for a message of bandwidth W is $N_0 W$
- For DSB-SC modulation the average modulated message power is given by

$$E[s^2(t)] = \frac{A_c^2 P}{2}$$

- Consequently the reference SNR for this transmission scheme is $SNR_{ref} = A_c^2 P / (2N_0 W)$

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{DSB}}}{\text{SNR}_{\text{ref}}} = 1$$

9.4: Noise in Linear Receivers Using Coherent Detection

Noise in DSB-SC modulation scheme

- This illustrates that we lose nothing in performance by using a band-pass modulation scheme compared to the baseband modulation scheme, even though the bandwidth of the former is twice as wide
- Consequently, DSB-SC modulation provides a ***baseline*** against which we may compare other amplitude modulation detection schemes.

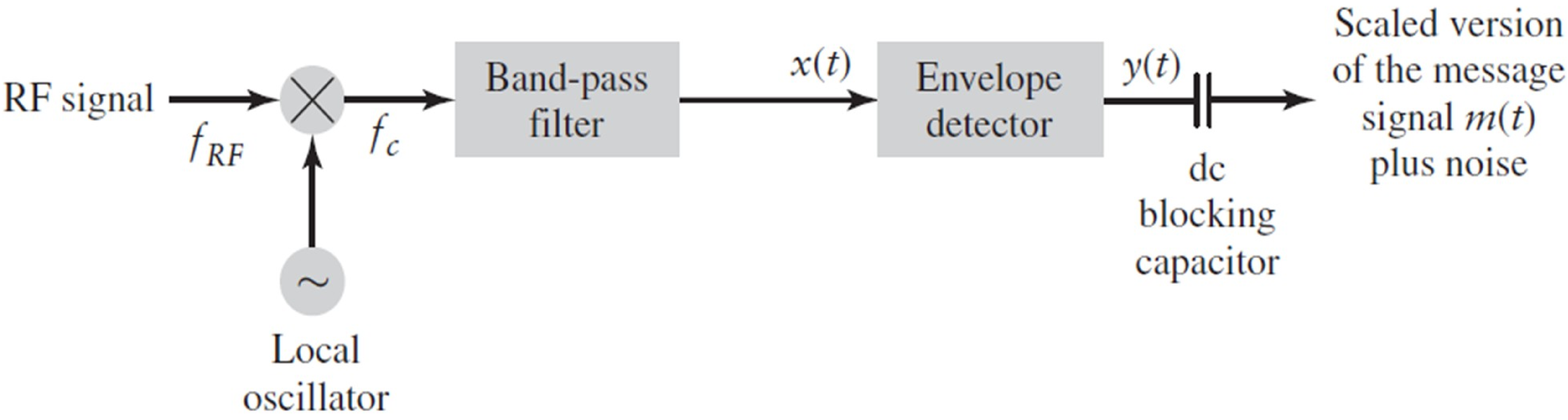
Section 9.5: Noise In AM Receivers Using Envelope Detection

9.5: Noise In AM Receivers Using Envelope Detection

- The envelope-modulated signal is represented by

$$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$

- Where $\boldsymbol{k_a}$ is the amplitude sensitivity of the modulator
- Model of AM receiver using envelope detection



9.5: Noise In AM Receivers Using Envelope Detection

PRE-DETECTION SNR

$$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$

- The average power of the carrier component is $A_c^2/2$ due to the sinusoidal nature of the carrier
- The power in the modulated part of the signal is

$$\begin{aligned} E[(1 + k_a m(t))^2] &= E[1 + 2k_a m(t) + k_a^2 m^2(t)] \\ &= 1 + 2k_a E[m(t)] + k_a^2 E[m^2(t)] \\ &= 1 + k_a^2 P \end{aligned}$$

Where we assume the message signal has zero mean

$$E[m(t)] = 0$$

9.5: Noise In AM Receivers Using Envelope Detection

PRE-DETECTION SNR

- Consequently, the received signal power is $A_c^2(1+k_a^2 P)/2$
- The ***pre-detection signal-to-noise ratio*** is given by

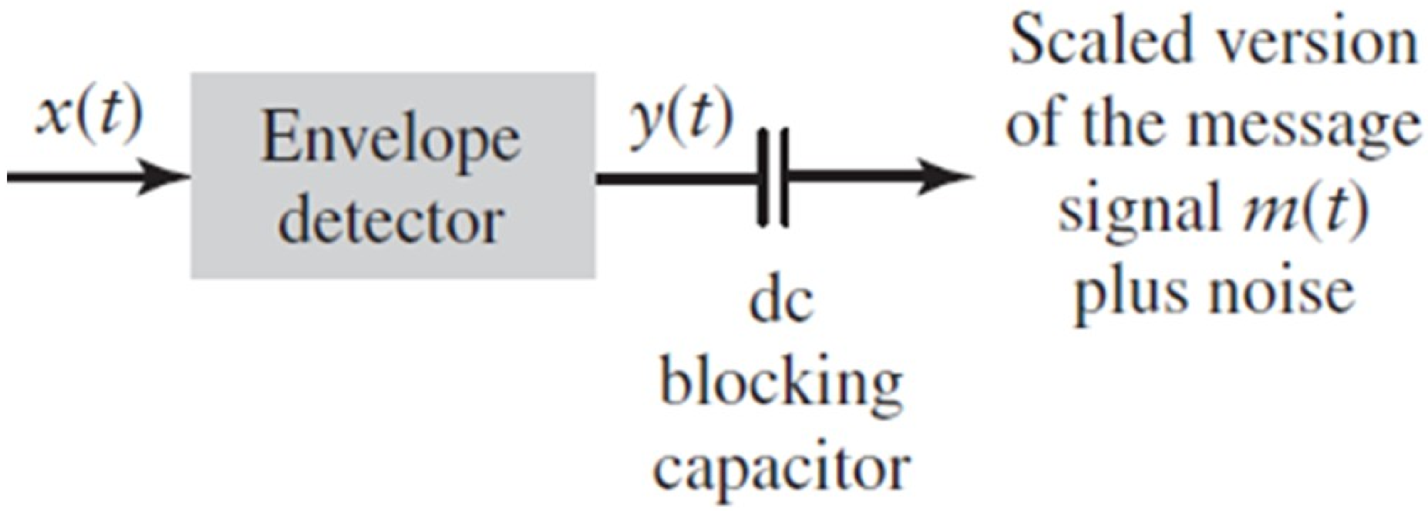
$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2N_0 B_T}$$

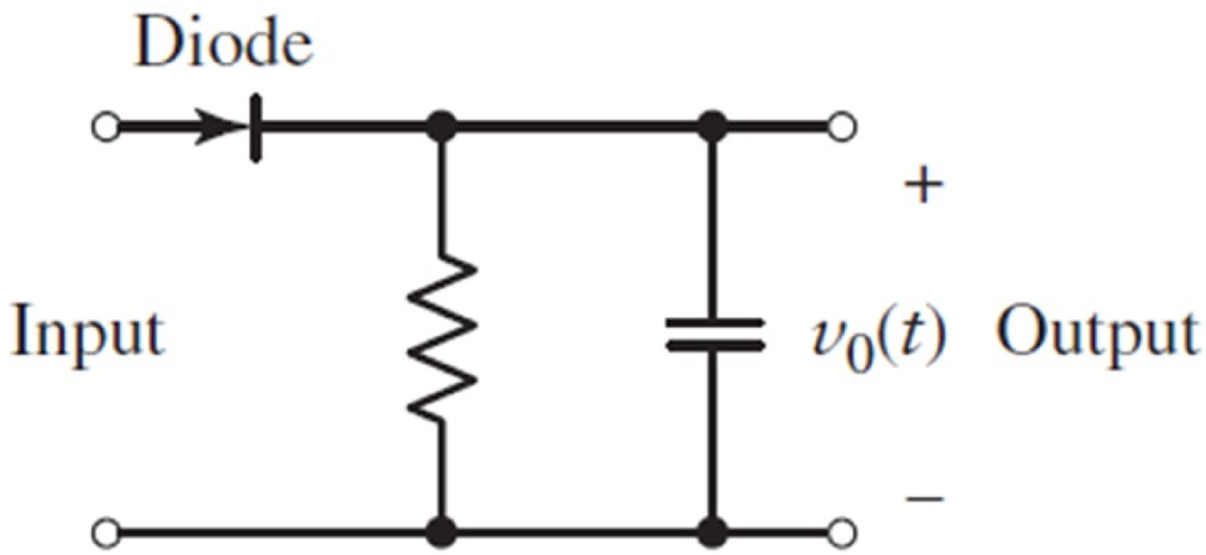
where B_T is the noise bandwidth of the band-pass filter

9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

- To determine the post-detection signal-to-noise ratio, we must analyze the effects of the remainder of the envelope detector





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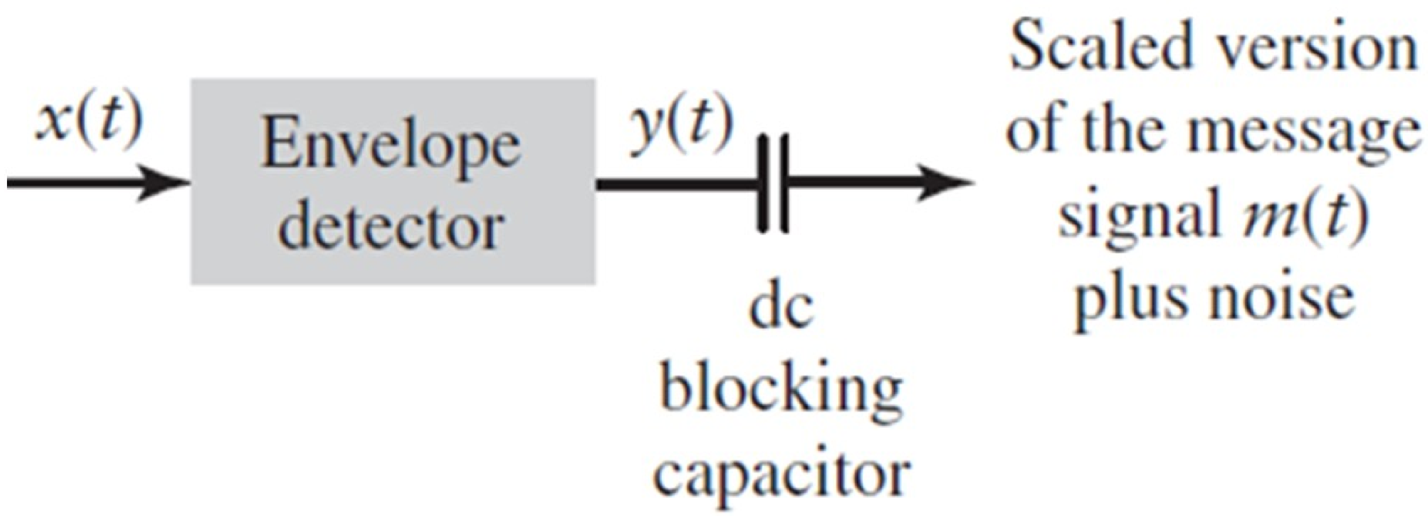
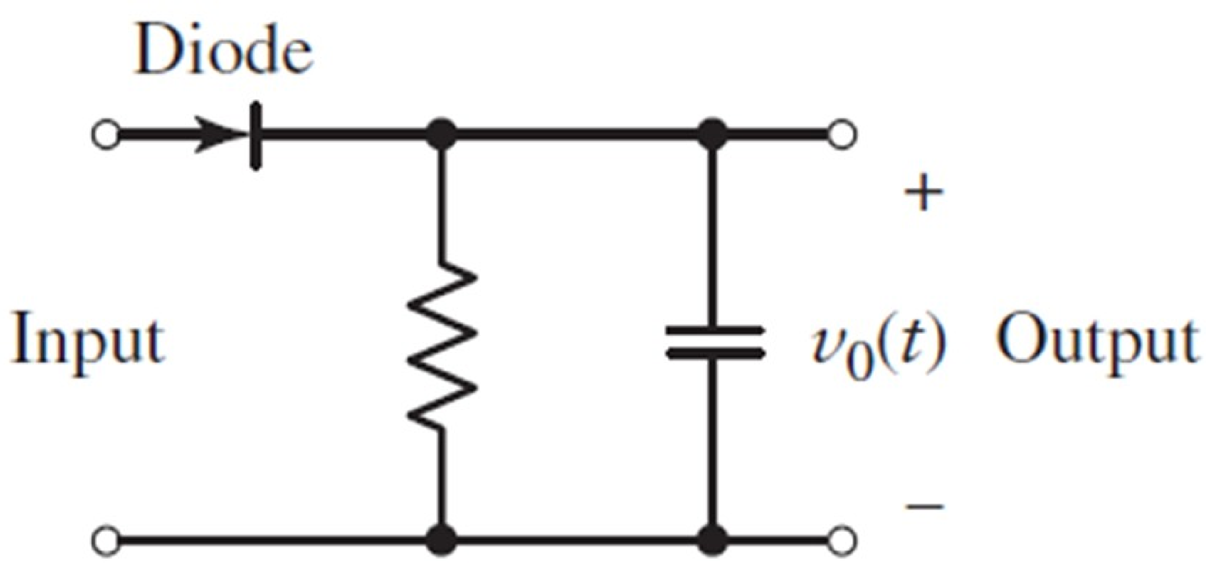
9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

- We can represent the noise in terms of its in-phase and quadrature components, and consequently model the input to the envelope detector as

$$x(t) = s(t) + n(t)$$
$$= [A_c + A_c k_a m(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$

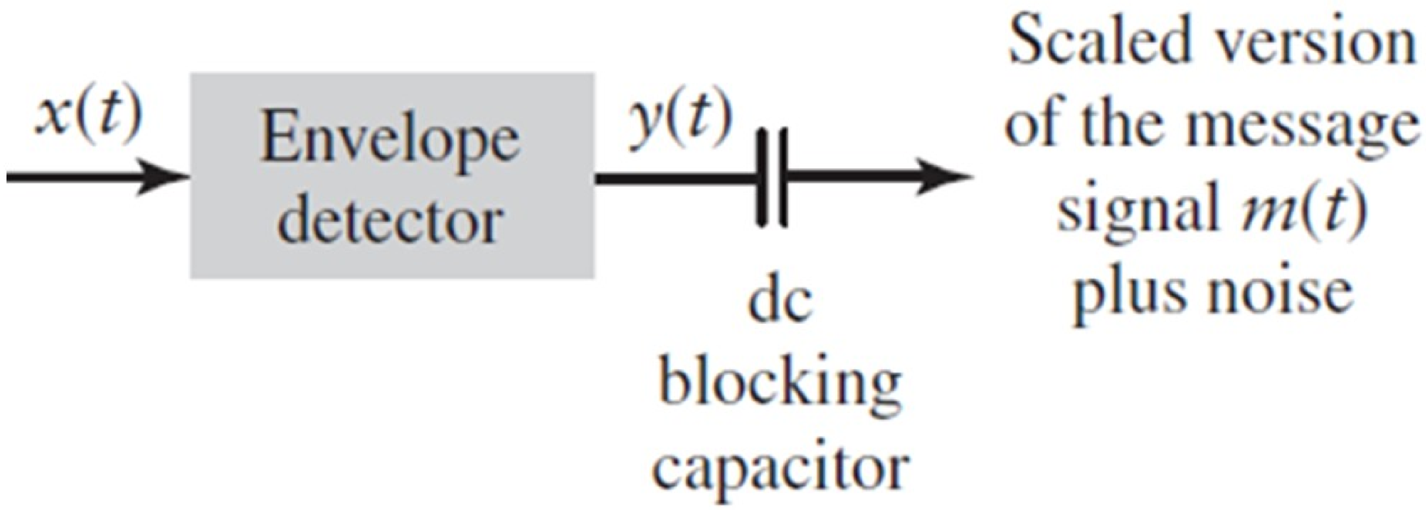


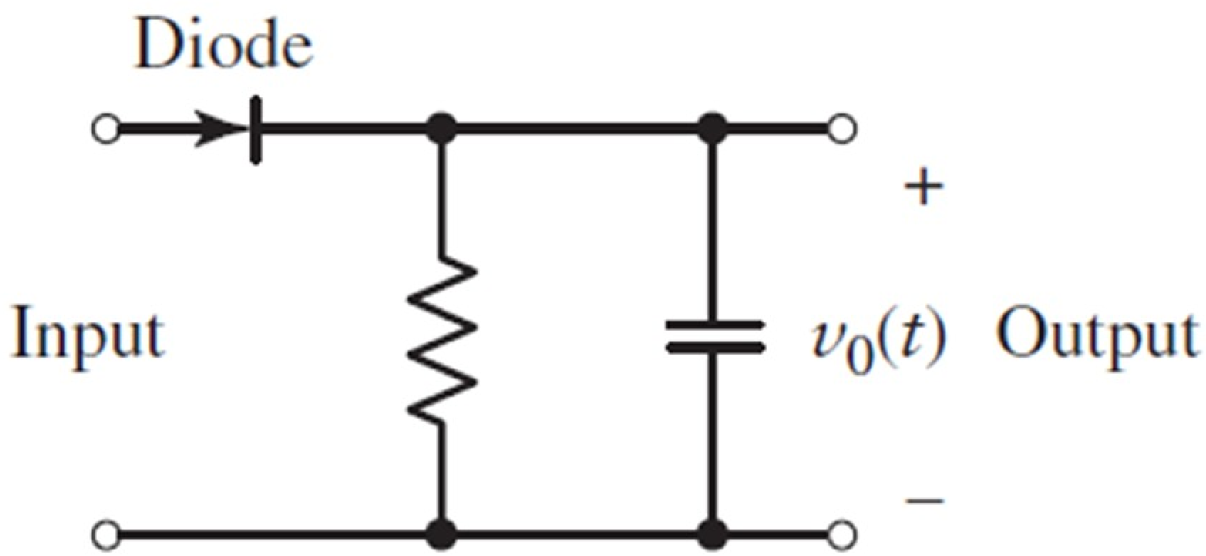


9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

- The object of the envelope detector is to recover the ***low-frequency amplitude*** variations of the ***high-frequency signal***





9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

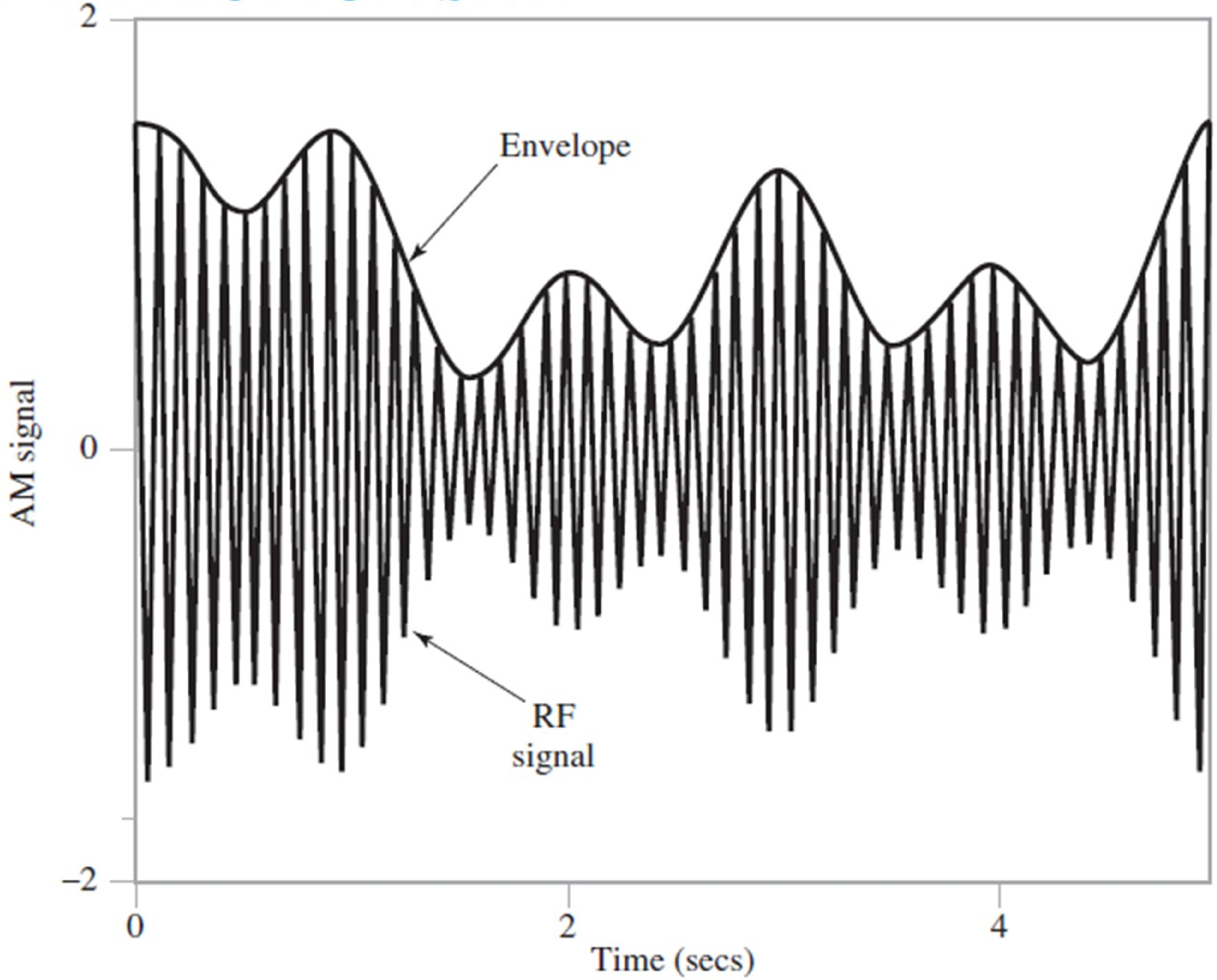


FIGURE 9.10 Illustration of envelope on high-frequency carrier.

9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

Using Phasor Diagram for representation

- The signal component of the phasor is $A_c(1+k_a m(t))$, and the noise has two orthogonal phasor components, $n_I(t)$ and $n_Q(t)$

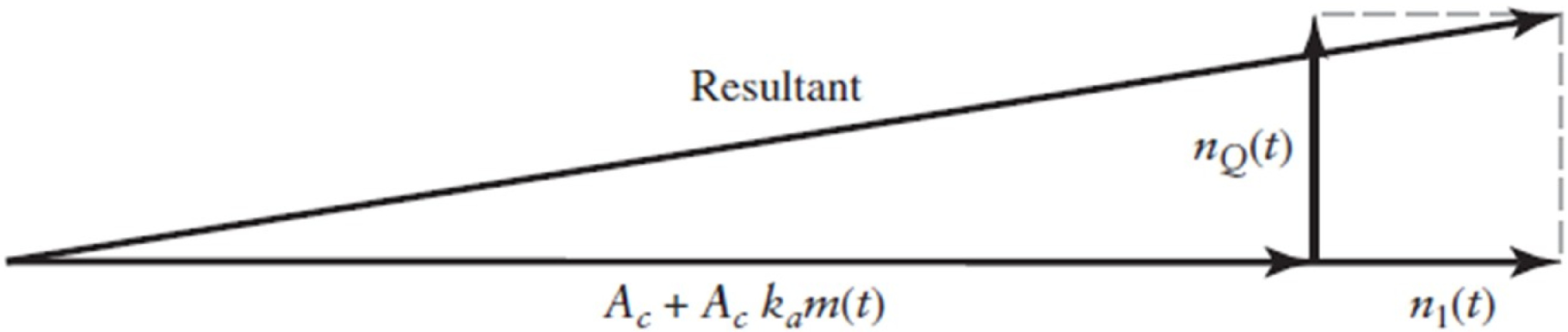


FIGURE 9.11 Phasor diagram for AM wave plus narrowband noise.

9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

- The output of the envelope detector is the amplitude of the phasor representing $x(t)$ and it is given by

$$y(t) = \text{envelope of } x(t)$$
$$= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$

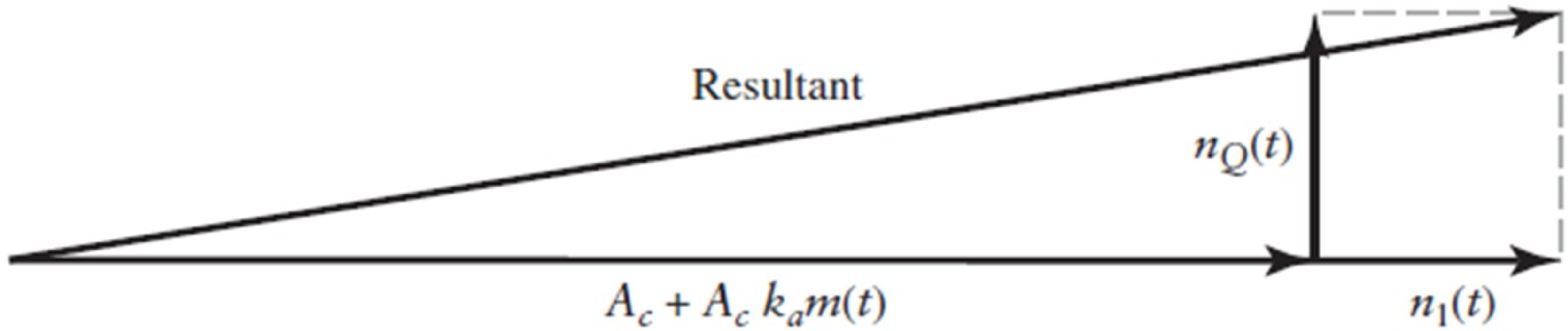


FIGURE 9.11 Phasor diagram for AM wave plus narrowband noise.

9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

$$y(t) = \text{envelope of } x(t)$$
$$= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$

If we assume that the signal is much larger than the noise then

- Using the approximation **√(A²+B²) ≈ A** when **A>>B**, we may write

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t)$$

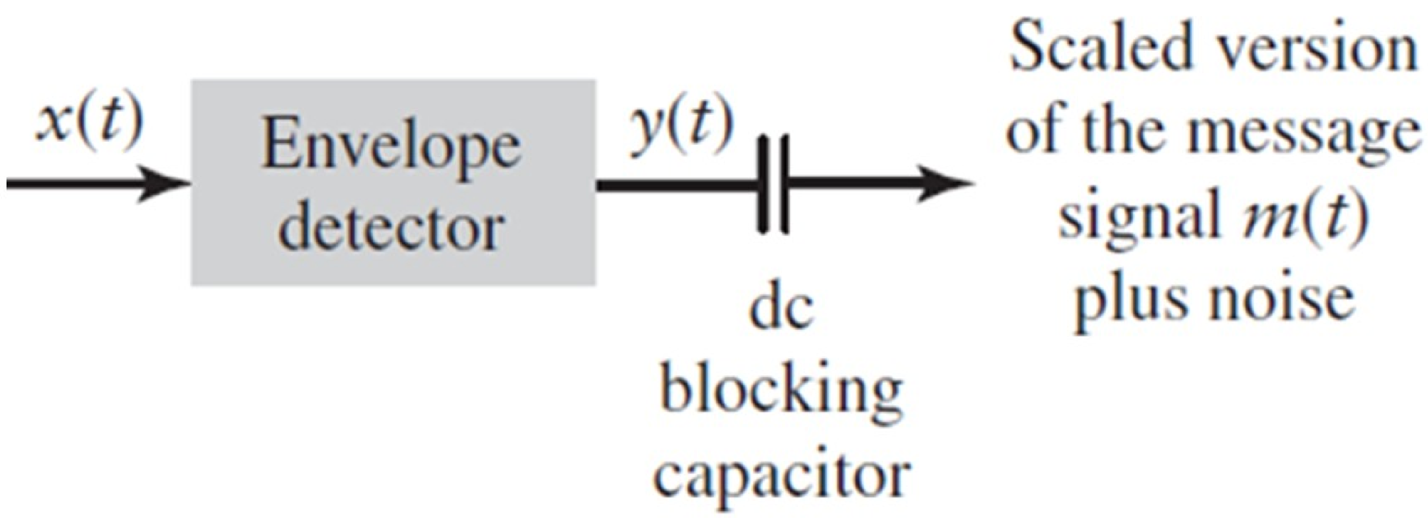
9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t)$$

This new expression for the demodulated signal has three components: ***dc component***, ***signal component***, and ***noise component***

- The dc term can be removed with a capacitor



9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

$$y(t) \approx A_c k_a m(t) + n_I(t)$$

Accordingly, the post-detection SNR for the envelope detection of AM, using a message bandwidth **W**, is given by

$$\text{SNR}^{\text{AM}}_{\text{post}} = \frac{A_c^2 k_a^2 P}{2N_0 W}$$

Where the numerator represents the average power of the message $\boldsymbol{A_c k_a m(t)}$ and the denominator represents the average power of $\boldsymbol{n_I(t)}$

9.5: Noise In AM Receivers Using Envelope Detection

POST-DETECTION SNR

$$\text{SNR}_{\text{post}}^{\text{AM}} = \frac{A_c^2 k_a^2 P}{2N_0 W}$$

This evaluation of the output SNR is only valid under two conditions:

1. The SNR is high (signal power is high compared with noise power).
2. k_a is adjusted for *100% modulation or less*, so there is no distortion of the signal envelope

- As with suppressed-carrier amplitude modulation, the message bandwidth **W** is approximately one-half of the transmission bandwidth $\mathbf{B_T}$

9.5: Noise In AM Receivers Using Envelope Detection

FIGURE OF MERIT

For AM modulation, the average transmitted power is given by

$(1+k_a^2 P)A_c^2/2$

- Consequently, the reference **SNR** is

$A_c^2(1+k_a^2 P)/(2N_0 W)$

- The figure of merit for this AM modulation– demodulation scheme is

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{AM}}}{\text{SNR}_{\text{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P}$$

9.5: Noise In AM Receivers Using Envelope Detection

FIGURE OF MERIT

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{AM}}}{\text{SNR}_{\text{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P}$$

- Since the product is always ***less than unity*** (otherwise the signal would be over modulated), the figure of merit for this system is ***always less than 0.5***
- Hence, the noise performance of an envelope-detector receiver is always lower than a DSB-SC receiver
- The reason is that at least half of the power is wasted transmitting the ***carrier as a component*** of the modulated (transmitted) signal.

Section 9.6: Noise in SSB Receivers

9.6: Noise in SSB Receivers

- We assume that only the *lower sideband* is transmitted, so that we may express the modulated wave as

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

Observations:

1. The two components m(t) and m^(t) are ***uncorrelated*** with each other. Therefore, their power spectral densities are ***additive***.
2. The Hilbert transform m^(t) is obtained by passing m(t) through a linear filter with transfer function ***–jsgn(f)***. The squared magnitude of this transfer function ***is equal to one*** for all Accordingly, ***m(t) and m^(t) have the same average power***

9.6: Noise in SSB Receivers

- Thus, proceeding in a manner similar to that for the DSB-SC receiver, we find that:
- The in-phase and quadrature components of the SSB modulated wave s(t) contribute an average power of $A_c^2 P / 8$ each.
- The average power of s(t) is therefore $A_c^2 P / 4$.
- This result is ***half*** that of the DSB-SC case, which is intuitively satisfying.

9.6: Noise in SSB Receivers

PRE-DETECTION SNR

- For the SSB signal, the transmission bandwidth B_T is approximately equal to the message bandwidth ***W***

- Consequently, using the signal power calculation of the previous section, the pre-detection signal-to-noise ratio of a coherent receiver with SSB modulation is

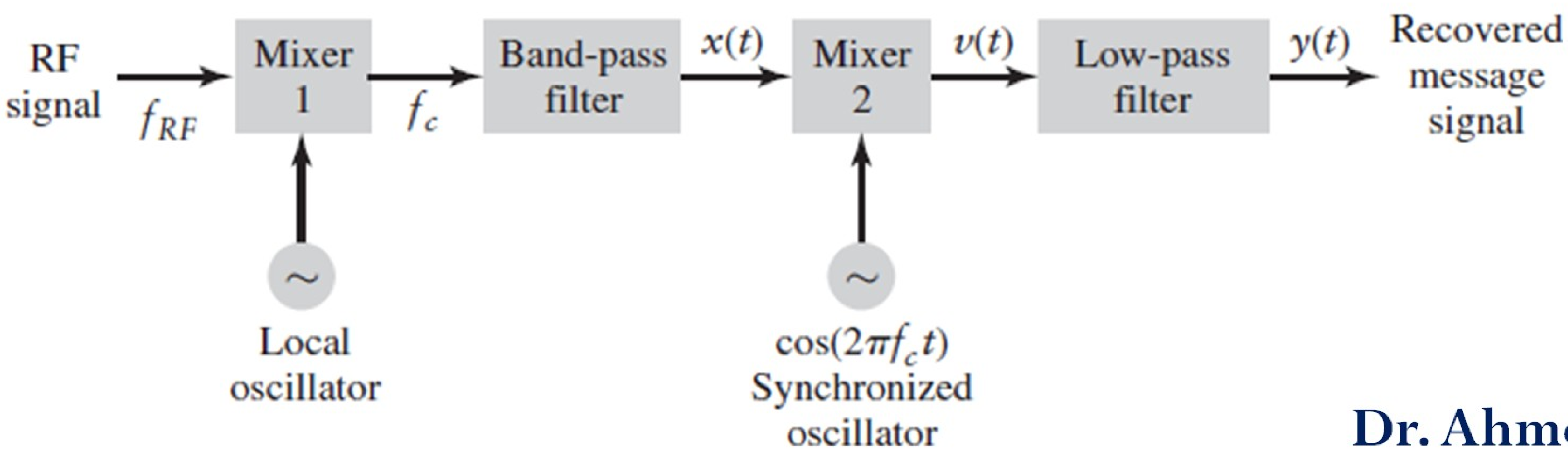
$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{SSB}} = \frac{A_c^2 P}{4N_0 W}$$

9.6: Noise in SSB Receivers

POST-DETECTION SNR

- Using the same superheterodyne receiver, the band-pass signal after multiplication with the synchronous oscillator output $\cos(2\pi f_c t)$ is

$$v(t) = x(t)\cos(2\pi f_c t)$$
$$= \frac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right) + \frac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right)\cos(4\pi f_c t) - \frac{1}{2}\left(\frac{A_c}{2}\hat{m}(t) + n_Q(t)\right)\sin(4\pi f_c t)$$



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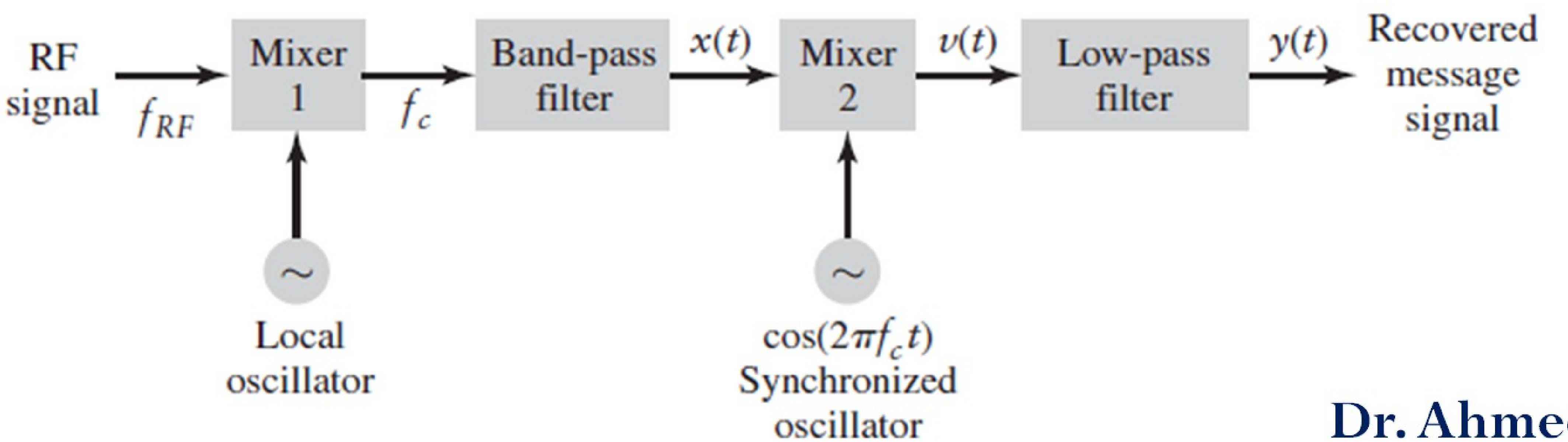
9.6: Noise in SSB Receivers

POST-DETECTION SNR

- After low-pass filtering the ***v***(t) we are left with

$$y(t) = \frac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right)$$

- As expected, we see that the quadrature component m^(t) of the message signal has been eliminated from the detector output



9.6: Noise in SSB Receivers

POST-DETECTION SNR

- The band-pass noise ***n(t)*** will also be of single sideband nature
- The spectrum of the in-phase component of the noise $\boldsymbol{n_I(t)}$ is given by (from section 8, Eq. (8.98))

$$S_{N_I}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & -B \leq f \leq B \\ 0, & \text{otherwise} \end{cases}$$

9.6: Noise in SSB Receivers

POST-DETECTION SNR

- For the single sideband case, $S_N(f)$ is $N_0/2$ for $f_c < f < f_c + W$ and for $-f_c - W < f < -f_c$, Consequently,

$$S_{N_I}(f) = \begin{cases} \frac{N_0}{2}, & -W \le f \le W \\ 0, & \text{otherwise} \end{cases}$$

- The spectral density of is double-sided, as in the DSB-SC case, but with half of the power

9.6: Noise in SSB Receivers

POST-DETECTION SNR

$$y(t) = \frac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right)$$

- The message component in the receiver output is $A_c m(t)/4$, for that the average power of the recovered message is $A_c^2 P/16$
- The corresponding noise power ¼ $N_0 W$

$$\text{SNR}_{\text{post}}^{\text{SSB}} = \frac{A_c^2 P}{4 N_o W}$$

9.6: Noise in SSB Receivers

FIGURE OF MERIT

- The average signal power for the SSB system, as discussed above, is $A_c^2 P/4$
- Consequently, the reference SNR is $A_c^2 P/(4N_0 W)$
- The figure of merit for the SSB system is the ratio of

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{SSB}}}{\text{SNR}_{\text{ref}}} = 1$$

➢ Consequently, SSB transmission has the same figure of merit as DSB-SC.

➢ The performance of vestigial sideband with coherent detection is similar to that of SSB.

9.6: Noise in SSB Receivers

Comparison:

- DSB-SC provides the same SNR performance as the baseband reference model but requires synchronization to perform coherent detection
- AM-LC simplifies the receiver design significantly as it is implemented with an envelope detector. However, AM-LC requires significantly more transmitter power to obtain the same SNR performance as the baseband reference model
- SSB-SC achieves the same SNR performance as the baseband reference model but only requires half the transmission bandwidth of the DSC-SC system. On the other hand, SSB requires more transmitter processing

Section 9.7: Detection of Frequency Modulation (FM)

9.7: Detection of Frequency Modulation (FM)

- Recall from Section 4.1 that the frequency-modulated signal is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right] \qquad (9.40)$$

- The received FM signal has a carrier frequency $\mathbf{f_c}$ and a transmission bandwidth $\mathbf{B_T}$ such that a negligible amount of power lies outside the frequency band $\mathbf{f_c} \pm \mathbf{B_T}/2$ for positive frequencies, and similarly for negative frequencies.

9.7: Detection of Frequency Modulation (FM)

PRE-DETECTION SNR

- For FM detection, we assume a receiver model as shown in

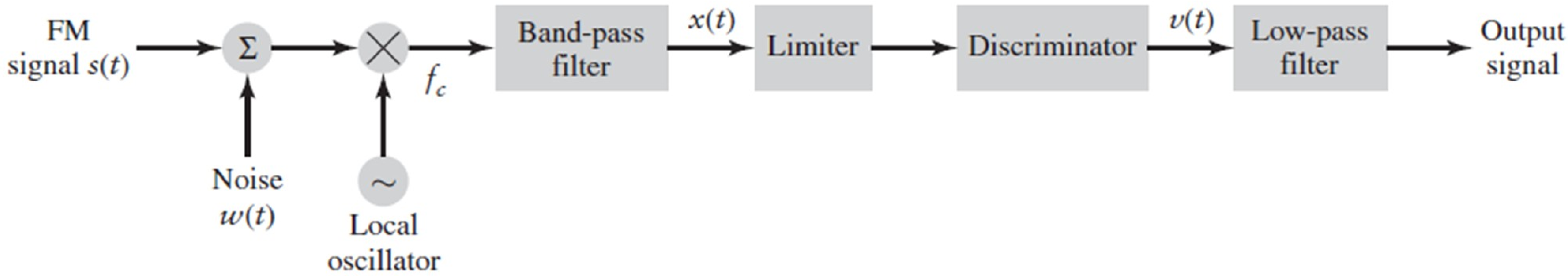


FIGURE 9.13 Model of an FM receiver.

- we assume that the noise is a white zero-mean Gaussian process with power spectral density $N_0/2$

9.7: Detection of Frequency Modulation (FM)

- The bandpass filter has a center frequency $\mathbf{f_c}$ and bandwidth $\mathbf{B_T}$ so that it passes the FM signal without distortion.
- Ordinarily, $\mathbf{B_T}$ is small compared with the center frequency so that we may use the narrowband **n(t)** representation for the filtered version of the channel noise **w(t)**

9.7: Detection of Frequency Modulation (FM)

- The ***pre-detection SNR*** in this case is simply the carrier power $\mathbf{A_c{}^2/2}$ divided by the noise passed by the bandpass filter, $\mathbf{N_0B_T}$; namely,

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} = \frac{A_c^2}{2N_0B_T}$$

9.7: Detection of Frequency Modulation (FM)

- In an FM system, the message signal is embedded in the variations of ***the instantaneous frequency*** of the carrier
- The amplitude of the carrier is ***constant***.
- ***Therefore*** any variations of the carrier amplitude at the receiver input must result from ***noise*** or ***interference***
- The amplitude ***limiter***, following the band-pass filter in the receiver model of Fig. 9.13, is used ***to remove amplitude variations by clipping the modulated wave***

9.7: Detection of Frequency Modulation (FM)

- The resulting wave is almost rectangular
- This wave is rounded by another band-pass filter that is an integral part of the limiter, thereby suppressing harmonics of the carrier frequency that are caused by the clipping.

9.7: Detection of Frequency Modulation (FM)

The discriminator in the model of Fig. 9.13 consists of two components

1. ***A slope network or differentiator:*** It produces a hybrid-modulated wave in which both amplitude and frequency vary in accordance with the message signal.
2. ***An envelope detector*** that recovers the amplitude variation and reproduces the message signal.

9.7: Detection of Frequency Modulation (FM)

- The post-detection filter, labeled "***low-pass filter***" in Fig. 9.13, has a bandwidth that is just large enough to pass the highest frequency component of the message signal

- This filter removes the out-of-band components of the noise at the discriminator output and thereby keeps the effect of the output noise to a minimum

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- The noisy FM signal after band-pass filtering may be represented as

$$x(t) = s(t) + n(t) \tag{9.41}$$

- We have expressed the filtered noise **n(t)** at the band-pass filter output in Fig. 9.13 in terms of its in-phase and quadrature components

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \tag{9.42}$$

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9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- We may equivalently express ***n(t)*** in terms of its envelope and phase as

$$n(t) = r(t)\cos[2\pi f_c t + \phi_n(t)] \qquad (9.43)$$

- where the envelope is

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2} \qquad (9.44)$$

- and the phase is

$$\phi_n(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \qquad (9.45)$$

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- To proceed, we note that the phase of s(t) is

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau \qquad (9.46)$$

- Combining Eqs. (9.40), (9.43), and (9.46), the noisy signal at the output of the band-pass filter may be expressed as

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \phi_n(t)] \end{aligned} \qquad (9.47)$$

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- It is informative to represent by means of a phasor diagram, as in Fig. 9.15 where we have used the signal term as the reference

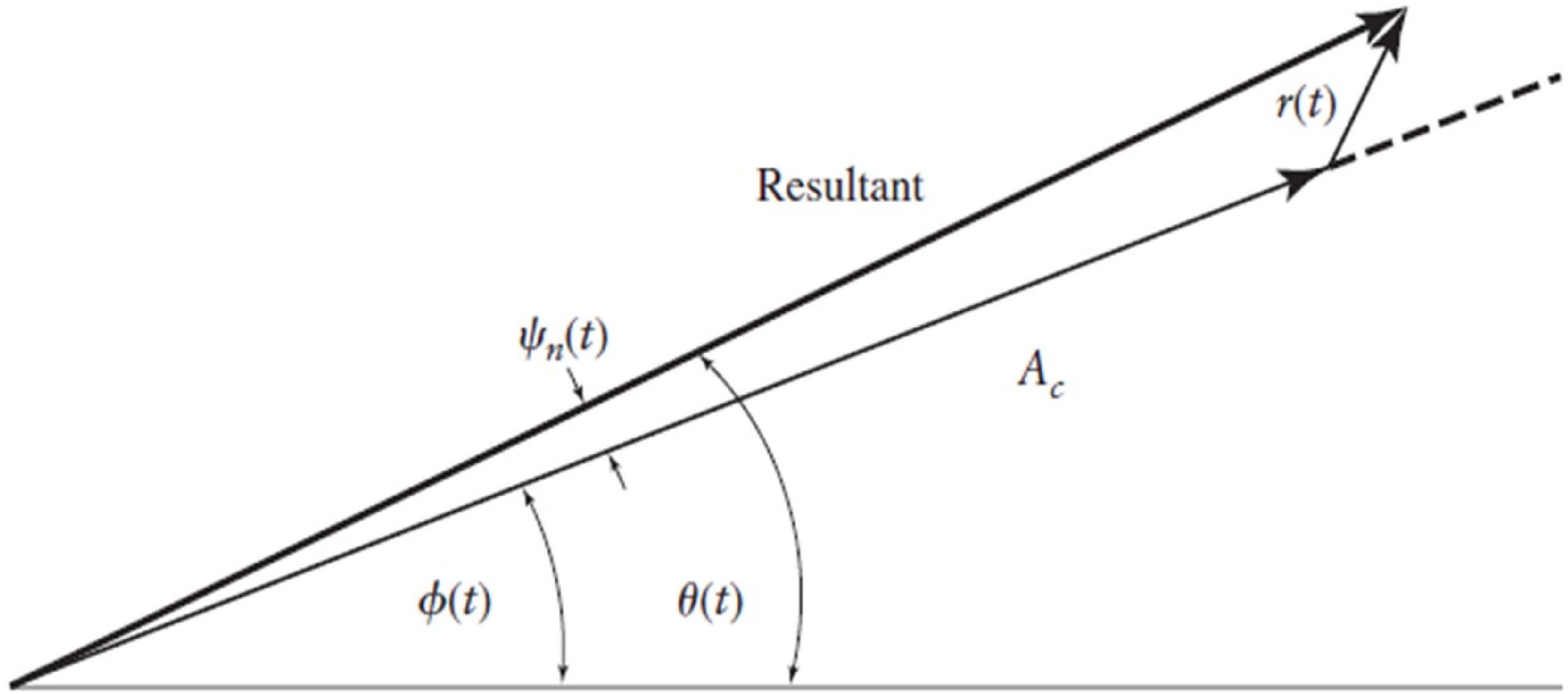

FIGURE 9.15 Phasor diagram for FM signal plus narrowband noise assuming high carrier-to-noise ratio.

$$x(t) = A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \phi_n(t)]$$

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- In Fig. 9.15, the amplitude of the noise is r(t) and the phase difference $\psi_n(t) = \phi_n(t) - \phi(t)$ is the angle between the noise phasor and the signal phasor.
- The phase $\theta(t)$ of the resultant is given by

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi_n(t))}{A_c + r(t)\cos(\psi_n(t))}\right\} \qquad (9.48)$$

- The envelope of x(t) is of no interest to us, because the envelope variations at the bandpass filter output ***are removed by the limiter***

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- To obtain useful results, we make some approximations regarding $\theta(t)$:
- As $\tan(x) \approx x$, as $x << 1$, the expression for the phase simplifies to

$$\theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin[\psi_n(t)] \qquad (9.49)$$

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9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- We simplify this expression even further by ignoring the modulation component in the second term of Eq. (9.49), and ***replacing $\psi_n(t) = \phi_n(t) - \phi(t)$ with $\phi_n(t)$,***
- because $\phi_n(t)$ is uniformly distributed between 0 and 2π radians and, since $\phi_n(t)$ is independent of $\phi(t)$, it is reasonable to assume that the phase difference is also uniformly distributed over radians 2π .

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- Then noting that the quadrature component of the noise is $n_Q(t) = r(t)\sin[\phi_n(t)]$ we may simplify Eq. (9.49) to

$$\theta(t) = \phi(t) + \frac{n_Q(t)}{A_c} \qquad (9.50)$$

- Using the expression for $\phi(t)$ given by Eq. (9.46), Eq. (9.50) can be expressed as

$$\theta(t) \approx 2\pi k_f \int_0^t m(\tau)\, d\tau + \frac{n_Q(t)}{A_c} \qquad (9.51)$$

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- *Our objective is to determine the error in the instantaneous frequency of the carrier wave caused by the presence of the filtered noise n(t)*
- With an ideal discriminator, its output is proportional to the derivative $d\theta(t)/dt$
- Using the expression for $\theta(t)$ in Eq. (9.51), the ideal discriminator output, scaled by 2π is therefore

$$
\begin{aligned}
v(t) &= \frac{1}{2\pi}\frac{d\theta(t)}{dt} \\
&= k_f m(t) + n_d(t) \qquad (9.52)
\end{aligned}
$$

9.7: Detection of Frequency Modulation (FM)

POST-DETECTION SNR

- where the noise term $n_d(t)$ is defined by

$$n_d(t) = \frac{1}{2\pi A_c} \frac{dn_Q(t)}{dt} \tag{9.53}$$

- We now see that, provided the carrier-to-noise ratio is high, the discriminator output $v(t)$ consists of the ***original message signal m(t)*** multiplied by the constant factor $\mathbf{k_f}$ plus an additive noise component $n_d(t)$

9.7: Detection of Frequency Modulation (FM)

The additive noise at the discriminator output is determined essentially by the quadrature component $n_Q(t)$ of the narrowband noise $n(t)$

- The ***post-detection signal-to-noise ratio*** is defined as the ratio of the average output signal power to the average output noise power

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9.7: Detection of Frequency Modulation (FM)

- From Eq. (9.52) we see that the message component of the discriminator output, and therefore the low-pass filter output, is $k_f m(t)$
- Hence, the average output signal power is equal to $k_f^2 P$ where P is the average power of the message signal m(t)
- To determine the average output noise power, we note that the noise $n_d(t)$ at the discriminator output is proportional to the time derivative of the quadrature noise component $n_Q(t)$

9.7: Detection of Frequency Modulation (FM)

- Since the differentiation of a function with respect to time corresponds to multiplication of its Fourier transform by $-j2\pi f$, it follows that we may obtain the noise process by passing $n_Q(t)$ through a linear filter with a frequency response equal to

$$G(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c} \tag{9.54}$$

9.7: Detection of Frequency Modulation (FM)

- This means that the power spectral density $S_{Nd}(f)$ of the noise $n_d(t)$ is related to the power spectral density $S_{NQ}(f)$ of the quadrature noise component $n_Q(t)$ as follows:

$$S_{N_d}(f) = |G(f)|^2 S_{N_Q}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f) \qquad (9.55)$$

$$S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| < \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (9.56)$$

For More information refer to textbook

$$S_{N_o}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| < W \\ 0, & \text{otherwise} \end{cases} \qquad (9.57)$$

9.7: Detection of Frequency Modulation (FM)

- ***The average output noise power is determined by integrating the power spectral density $S_{N0}(f)$ from –W to W.***
- Doing so, we obtain the following result:

$$\text{Average post-detection noise power} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\, df = \frac{2N_0 W^3}{3A_c^2} \qquad (9.58)$$

9.7: Detection of Frequency Modulation (FM)

- Thereby,

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \tag{9.59}$$

- Hence, the post-detection SNR of an FM demodulator has a nonlinear dependence on both ***the frequency sensitivity*** and ***the message bandwidth***.

9.7: Detection of Frequency Modulation (FM)

Figure of Merit:

- With FM modulation, the modulated signal power is simply $\mathbf{A_c^2/2}$, hence the reference SNR is $\mathbf{A_c^2/(2N_0W)}$ Consequently, the figure of merit for this system is given by

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}} = \frac{\dfrac{3A_c^2k_f^2P}{2N_0W^3}}{\dfrac{A_c^2}{2N_0W}}$$

$$= 3\left(\frac{k_f^2P}{W^2}\right)$$

$$= 3D^2 \qquad (9.60)$$

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9.7: Detection of Frequency Modulation (FM)

$$\text{Figure of merit} = 3D^2$$

- Where, in the last line, we have introduced the definition $\mathbf{D=k_fP^{1/2}/W}$ as modulation index and so $\mathbf{k_fP^{1/2}}$ is ***the deviation ratio*** for the FM system in light of the material presented in Section 4.6.
- Recall from that section that the generalized Carson rule yields the transmission bandwidth for an FM signal

$$B_T = 2(k_f P^{1/2} + W) \approx 2k_f P^{1/2}$$

9.7: Detection of Frequency Modulation (FM)

- So, substituting $B_T/2$ for $\mathbf{k_f P^{1/2}}$ in the definition of **D**, the figure of merit for an FM system is approximately given by

$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{W}\right)^2 \tag{9.61}$$

9.7: Detection of Frequency Modulation (FM)

$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{W}\right)^2 \qquad (9.61)$$

- Consequently, an increase in the transmission bandwidth provides a corresponding quadratic increase in the output signal-to-noise ratio with an FM system compared to the reference system
- Thus, when the carrier to noise level is high, unlike an amplitude modulation system an FM system allows us to trade bandwidth for improved performance in accordance with a square law.