From the previous we can define a dimensionless quantity known as the modulation index

$$m = \frac{\text{peak amplitude of DSB-SC}}{\text{Peak Carrier amplitude}} = \frac{|\min(f(t))|}{A}$$

recall that from previous lecture we have

$$\phi_{AM}(t) = f(t)\cos w_c t + A\cos w_c t$$

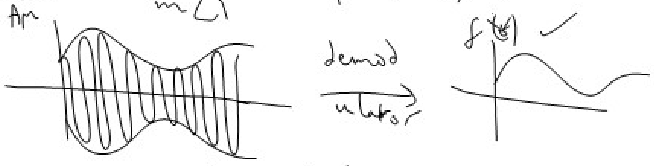
* If $f(t) = a_m \cos w_m t$, then

$$\phi_{AM}(t) = a_m \cos w_m t \cos w_c t + A\cos w_c t$$
$$= A\left[1 + \frac{a_m \cos w_m t}{A}\right]\cos w_c t$$
$$= A\left[1 + m\cos w_m t\right]\cos w_c t$$

* If we consider the previous three modulation conditions which are

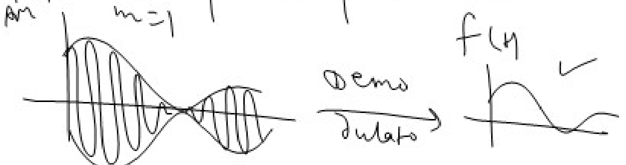
1. $A > |\min f(t)| \longrightarrow 1 > m$

$\phi_{AM}(t)$ $m < 1$



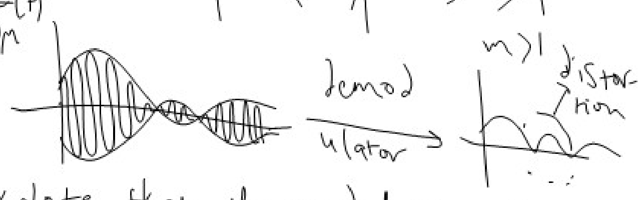
2. $A = |\min f(t)| \longrightarrow m = 1$

$\phi_{AM}(t)$ $m = 1$

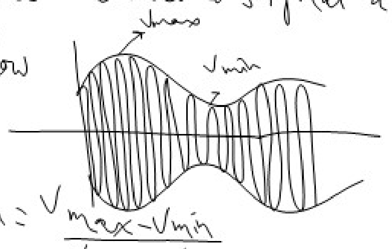


3. $A < |\min(f(t))| \longrightarrow m > 1$

$\phi_{AM}(t)$



* Note that the modulation index can be measured from the envelope of the modulated signal as shown below



$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$V_{max} = A + a_m = A(1+m)$$
$$V_{min} = A - a_m = A(1-m)$$

$$m = \frac{A(1+m) - A(1-m)}{A(1+m) + A(1-m)} = \frac{2Am}{2A}$$

5.2.1 Carrier and Sideband power in AM

* The average power in AM modulated signal is given by

$$\overline{\phi^2_{AM}(t)} = \overline{\left[f(t)\cos w_c t + A\cos w_c t\right]^2}$$
$$= \overline{(f(t)\cos w_c t)^2} + \cancel{\overline{2A f(t)\cos^2 w_c t}} + \overline{A^2\cos^2 w_c t}$$
$$= \underbrace{\tfrac{1}{2}\overline{f^2(t)}}_{P_s} + \underbrace{\frac{A^2}{2}}_{P_c}$$

∴ The total transmitted power P_t is

$$P_t = P_c + P_s$$

The efficiency of AM modulated signal is

$$\mu = \frac{P_s}{P_t} = \frac{P_s}{P_c + P_s} = \frac{\tfrac{1}{2}\overline{f^2(t)}}{A^2/2 + \tfrac{1}{2}\overline{f^2(t)}}$$
$$= \frac{\overline{f^2(t)}}{A^2 + \overline{f^2(t)}}$$

If $f(t) = a_m \cos \omega_m t$, then

$$\overline{f^2(t)} = \frac{a_m^2}{2}$$

$$\mu = \frac{a_m^2/2}{A^2 + a_m^2/2}$$

$$= \frac{a_m^2}{2A^2 + a_m^2}$$

$$= \frac{a_m^2}{a_m^2\left[2\left(\frac{A}{a_m}\right)^2 + 1\right]} = \frac{1}{\frac{2}{m^2}+1}$$

$$= \frac{m^2}{m^2+2}$$

* The maximum allowed value for $m = 1$

$$\therefore \mu = \frac{1}{1+2} = 33\%$$