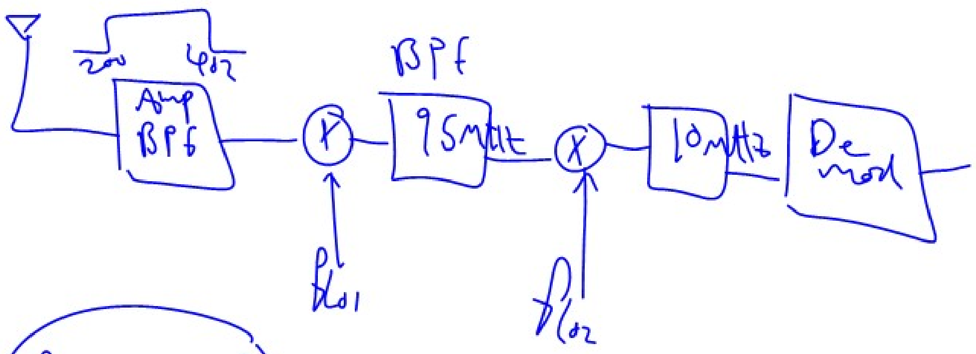
Double Conversion superhetrodyne receiver

200 – 402 MHz

BW = 4 MHz

$f_{IF_1} = 95\,MHz$, $f_{IF_2} = 10\,MHz$



$f_{lo1} > f_c$

$f_{lo1} = [200+95,\ 402+95]$

$[295,\ 497]$

$f_{lo2} = 95+10 = 105\,MHz$

$f_{IF} \geq \frac{402-200}{2} \geq 101$, Yes, there are images

$f_{im1} = 200+2(95) = 390$

$f_{im2} = 204+2(95) = 394$

$f_3 = 208+2(95) = 398$

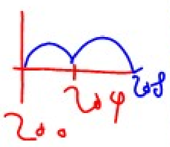
$f_4 = 212+2(95) = 402$

$f_5 = 402-2(95) = 212$

$f_6 = 398-2(95) = 208$

$f_7 = 394-2(95) = 204$

$f_8 = 390-2(95) = 200$



freq	U-SSB (W)	L-SSB (W)	DSB-SC (2W)
200	✓	NA	NA
204	✓	✓	✓
208	✓	✓	NA
212	✓	✓	✓
402	NA	✓	NA
398	✓	✓	✓
394	✓	✓	NA
390	✓	✓	✓

$\geq 101\,MHz$

Q1: $s(t) = 25[1 + 0.5\cos(2000\pi t)]\cos(40\times10^3\pi t)$

AM-LC

$2\pi f_m t$

$$s(t) = \underline{A_c}[1 + \underline{\mu}\cos(2\pi \underline{10^3}t)]\cos(2\pi \underline{(20\times10^3)}t)$$

A_c ; f_m ; $\mu = k_a A_m$, $A_m \rightarrow 2$; f_c

$$P_c = \frac{1}{2}A_c^2 = 312.5\ W$$

$$k_a = \frac{0.5}{2} = 0.25$$

$$\mu = 0.5$$

Communications and Signals Processing

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2012/2013

Chapter 4 - Outlines

Outlines

Introduction - *Angel modulation*

Angel modulation

The angle of the carrier wave is varied according to the information-bearing signal and the ***amplitude remains constant***

Lesson 1 : Angle modulation is a nonlinear process, which testifies to its sophisticated nature. In the context of analog communications, this distinctive property of angle modulation has two implications :

- In analytic terms, the spectral analysis of angle modulation is complicated
- In practical terms, the implementation of angle modulation is demanding

Introduction - *Angel modulation*

Lesson 2 : Whereas the transmission bandwidth of an amplitude-modulated wave is of limited extent, the transmission bandwidth of an angle-modulated wave may an infinite extent, at least in theory

Lesson 3 : Given that the amplitude of the carrier wave is maintained constant, we would intuitively expect that additive noise would affect the performance of angle modulation to a lesser extent than amplitude modulation.

Introduction - *Angel modulation*

- Angle modulation can provide better discrimination against noise and interference than amplitude modulation
- Angle modulation provides us with a practical means of ***exchanging channel bandwidth for improved noise performance***

Chapter 4.1- *Angel modulation*

Basic Definitions

Consider again the general carrier

$$V_c(t) = A_c \cos(\omega_c t + \phi_c)$$

$(\omega_c t + \phi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

a) By varying the frequency, ω_c – **Frequency Modulation.**

b) By varying the phase, ϕ_c – **Phase Modulation**

Chapter 4.1- *Angel modulation*

Basic Definitions – one way to derive FM modulation

In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Consider the carrier

$$v_c(t) = A_c \cos(\omega_c t)$$

then for FM we may write:

FM signal

$$v_s(t) = A_c \cos\left(2\pi\left(f_c + \text{frequency deviation}\right) \quad t\right)$$

,where the frequency deviation will depend on $m(t)$.

Chapter 4.1- *Angel modulation*

Basic Definitions

Given that the carrier frequency will change we may write for an instantaneous carrier signal

$$A_c \cos(\omega_i t) = A_c \cos(2\pi f_i t) = A_c \cos(\theta_i)$$

where θ_i is the instantaneous angle $= \omega_i t = 2\pi f_i t$ and f_i is the instantaneous frequency.

Chapter 4.1- *Angel modulation*

Basic Definitions

Since $\theta_i = 2\pi f_i t$ then $\dfrac{d\theta_i}{dt} = 2\pi f_i$ or $f_i = \dfrac{1}{2\pi}\dfrac{d\theta_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\theta_i}{dt}$$

Δf_c is the peak deviation of the carrier.

Hence, we have $\dfrac{1}{2\pi}\dfrac{d\theta_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$

,i.e. $\dfrac{d\theta_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$

Chapter 4.1- *Angel modulation*

Basic Definitions

After integration *i.e.* $\int(\omega_c + 2\pi\Delta f_c\cos(\omega_m t))dt$

$$\theta_i = \omega_c t + \frac{2\pi\Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\theta_i = \omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = A_c\cos(\theta_i)$

$$v_s(t) = A_c\cos\left(\omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)\right)$$

Chapter 4.1- *Angel modulation*

Basic Definitions

The ratio $\frac{\Delta f_c}{f_m}$ is called the Modulation Index denoted by β i.e.

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

- Note – FM, as implicit in the FM equation for $v_s(t)$, is a non-linear process
- The FM signal for a message m(t) as a band of signals is very complex. Hence, m(t) is usually considered as a 'single tone modulating signal' of the form

$$m(t) = A_m \cos(\omega_m t)$$

Chapter 4.1- *Angel modulation*

Basic Definitions – Other way to get s(t) for FM and PM

Angle-modulated wave

$$s(t) = A_c \cos[\theta_i(t)] \quad (4.1)$$

Where

- $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier at time t; it is assumed to be a function of the information-bearing signal or message signal
- A_c is the carrier amplitude

Chapter 4.1- *Angel modulation*

Basic Definitions – Other way to get s(t)

If $\theta_i(t)$ increases with time, then the ***average frequency*** in hertz, over a small interval from t to $t + \Delta t$, is given by

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

Chapter 4.1- *Angel modulation*

Basic Definitions – Other way to get s(t)

Allowing the time interval Δt to approach zero leads to the following definition for the *instantaneous frequency* of the angle-modulated signal s(t)

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[\frac{\theta_t(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right]$$

$$= \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \qquad (4.2)$$

$$\theta_i(t) = 2\pi f_c t + \phi_c, \quad \text{for } m(t) = 0$$

Chapter 4.1- *Angel modulation*

Basic Definitions – for general m(t)

1. Phase modulation (PM) is that form of angle modulation in which the instantaneous angle is varied linearly with the message signal

$$\theta_i(t) = 2\pi f_c t + k_p m(t) \quad (4.3)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \quad (4.4)$$

2. Frequency modulation (FM) is that form of angle modulation in which the instantaneous frequency is varied linearly with the message signal

$$f_i(t) = f_c + k_f m(t) \quad (4.5)$$

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \quad (4.6)$$

$$\frac{d\theta_i}{dt} = 2\pi f_i \quad \text{or} \quad f_i = \frac{1}{2\pi}\frac{d\theta_i}{dt}$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right] \quad (4.7)$$

Chapter 4.1- *Angel modulation*

Basic Definitions

TABLE 4.1 ***Summary of Basic Definitions in Angle Modulation***

	Phase modulation	*Frequency modulation*	*Comments*
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$	A_c: carrier amplitude f_c: carrier frequency $m(t)$: message signal k_p: phase-sensitivity factor k_f: frequency-sensitivity factor
	$\frac{d\theta_i}{dt} = 2\pi f_i$ or	$f_i = \frac{1}{2\pi}\frac{d\theta_i}{dt}$	
Instantaneous frequency $f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)$	$f_c + k_f m(t)$	
Modulated wave $s(t)$	$A_c \cos[2\pi f_c t + k_p m(t)]$	$A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$	

Chapter 4.2: Properties of Angle-Modulated Waves

Fig 4.1

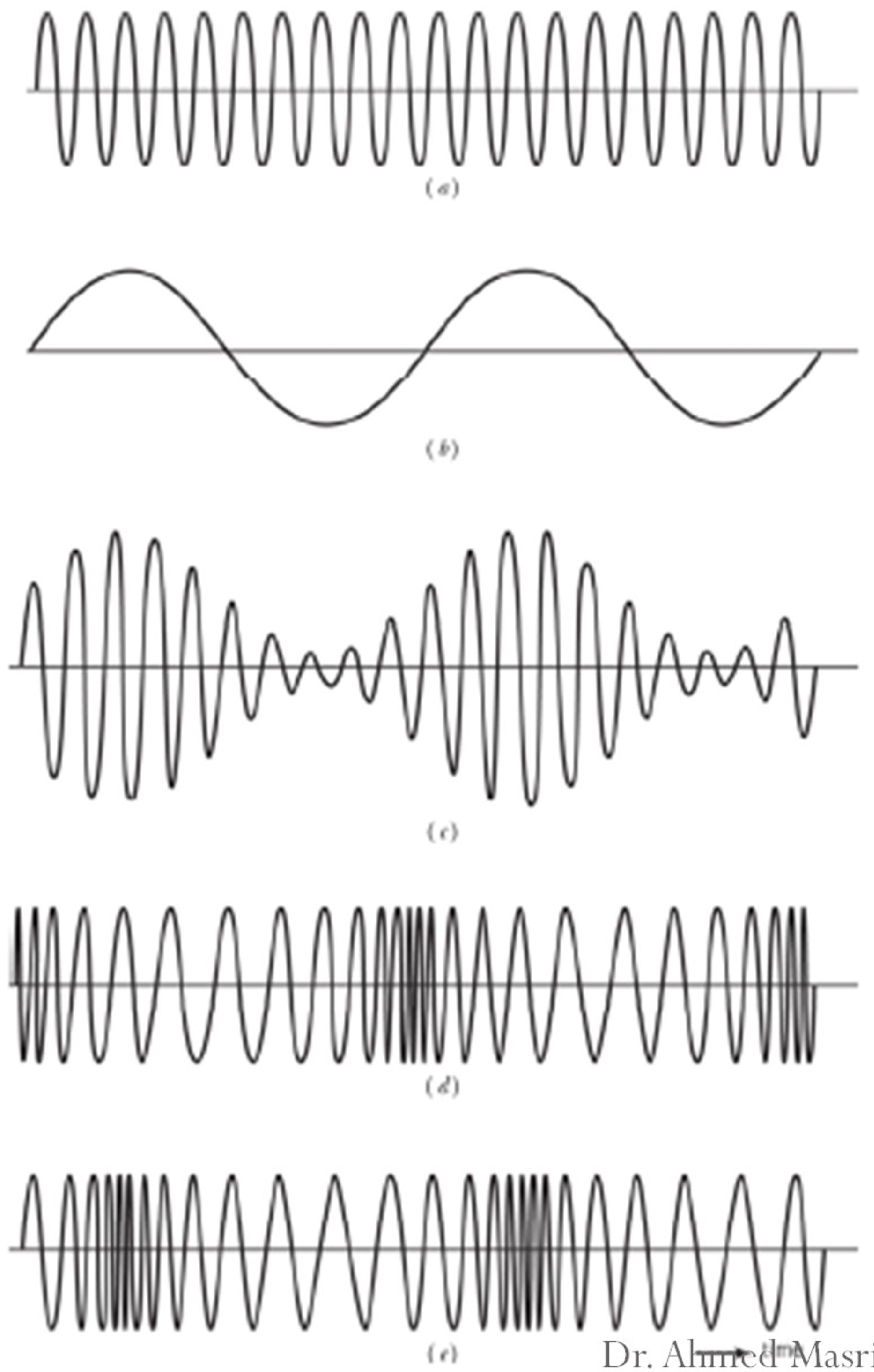
a) Carrier wave

b) Sinusoidal modulating signal

c) Amplitude-modulated signal

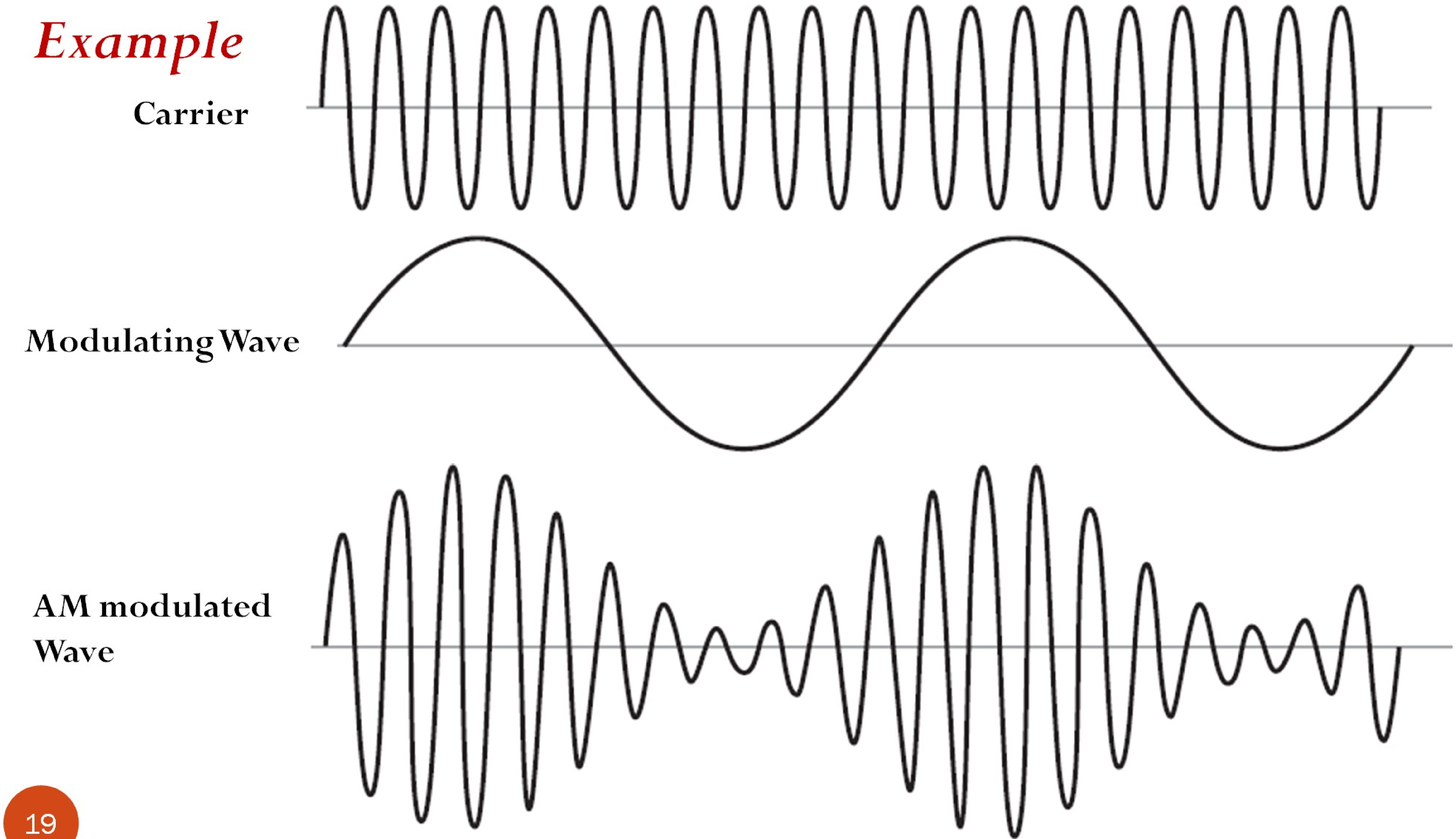
d) Phase-modulated signal

e) Frequency-modulated signal



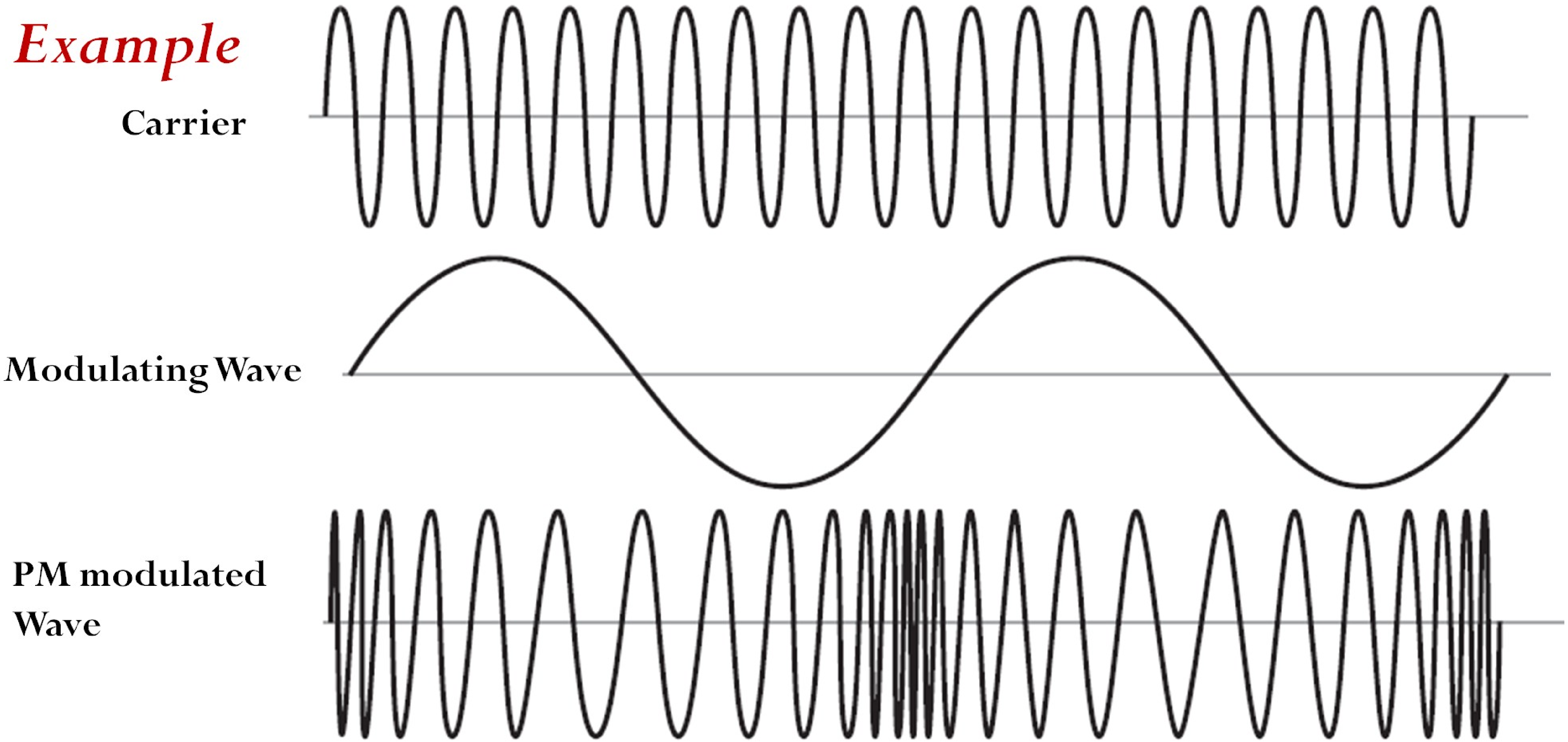
Chapter 4.2: Properties of Angle-Modulated Waves

Example



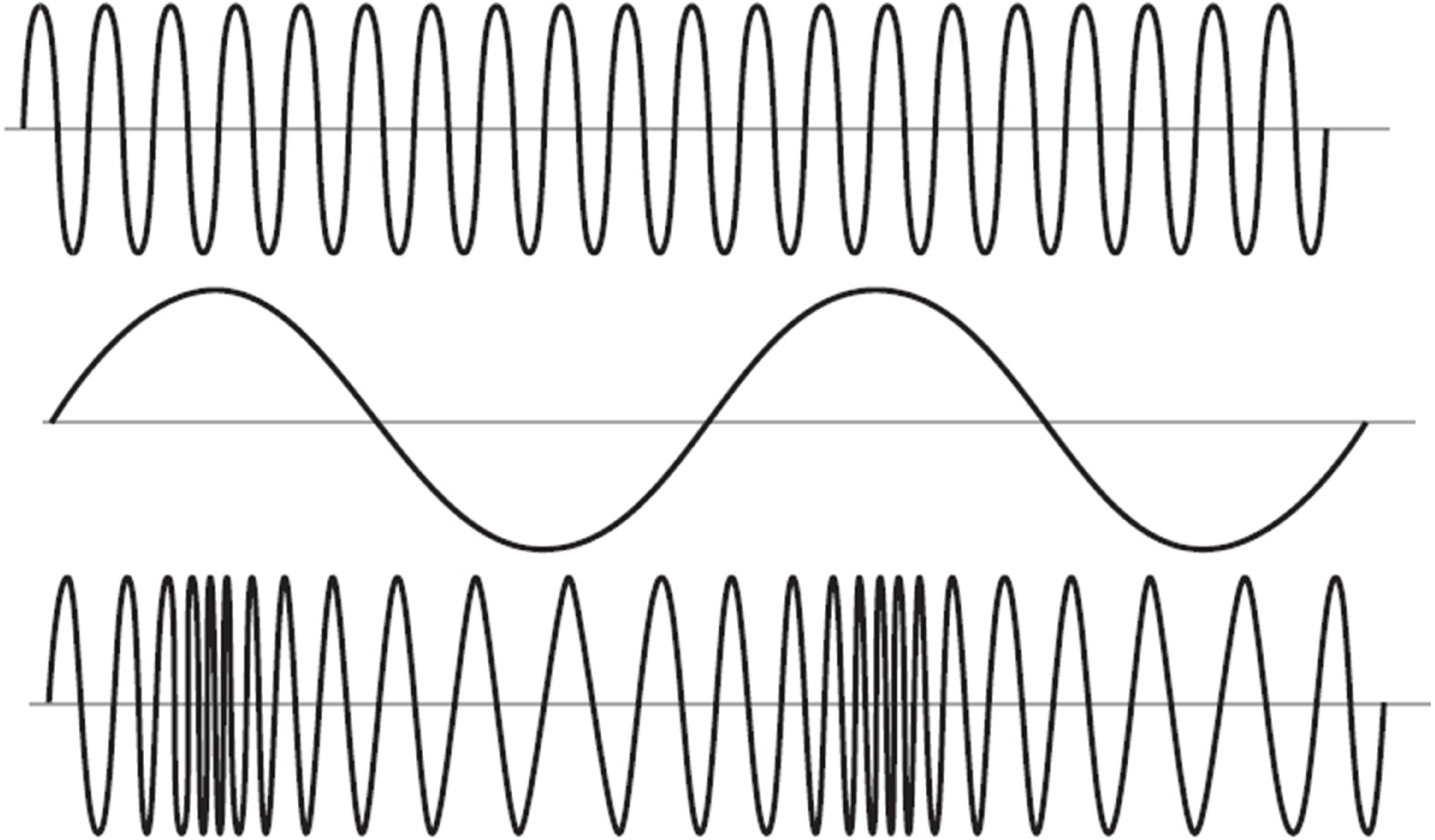
Chapter 4.2: Properties of Angle-Modulated Waves

Example



Chapter 4.2: Properties of Angle-Modulated Waves

Example



Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 1 Constancy of transmitted power

- The amplitude of PM and FM waves is maintained at a constant value equal to the carrier amplitude for all time.
- The average transmitted power of angle-modulated waves is a constant

$$P_{av} = \frac{1}{2} A_c^2 \qquad (4.8)$$

where it is assumed that the load resistor is 1 ohm.

$$\left(P = \frac{V^2}{R} \right)$$

Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 2 Nonlinearity of the modulation process

- We say so because both PM and FM waves violate the principle of superposition

Suppose, for example, that the message signal m(t) is made up of two different components m1(t) and m2(t) as shown by

$$m(t) = m_1(t) + m_2(t)$$

Then ,let s(t), s1(t) and s2(t) denote the PM waves as follow

$$s(t) = A_c \cos[2\pi f_c t + k_p(m_1(t) + m_2(t))]$$

$$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)]$$

$$s_2(t) = A_c \cos[2\pi f_c t + k_p m_2(t)]$$

$$s(t) \neq s_1(t) + s_2(t)$$

Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 2 Nonlinearity of the modulation process

- The fact that the angle-modulation process is nonlinear complicates the spectral analysis and noise analysis of PM and FM waves, compared to amplitude modulation

Chapter 4.2: Properties of Angle-Modulated Waves

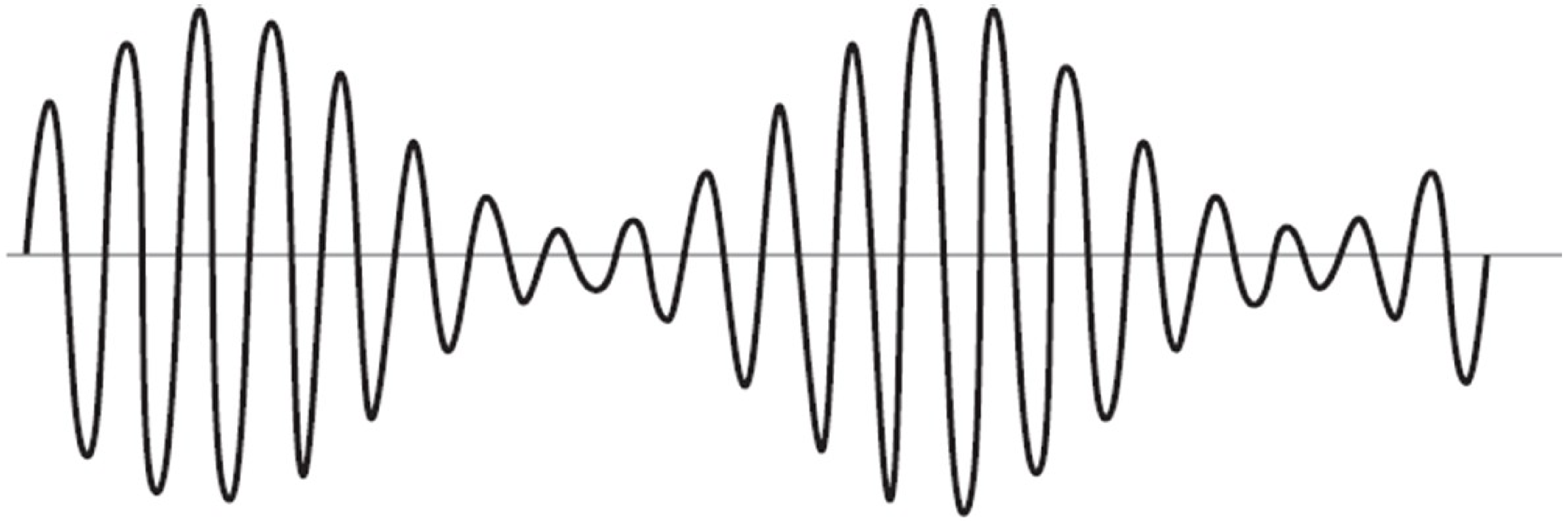
PROPERTY 3 Irregularity of zero-crossings

- *Zero-crossings* are defined as the instants of time at which a waveform changes its amplitude from a positive to negative value or the other way around.
- The zero-crossings of a PM or FM wave no longer have a perfect regularity in their spacing across the time-scale.
- The irregularity of zero-crossings in angle-modulation waves is also attributed to ***the nonlinear character of the modulation process.***

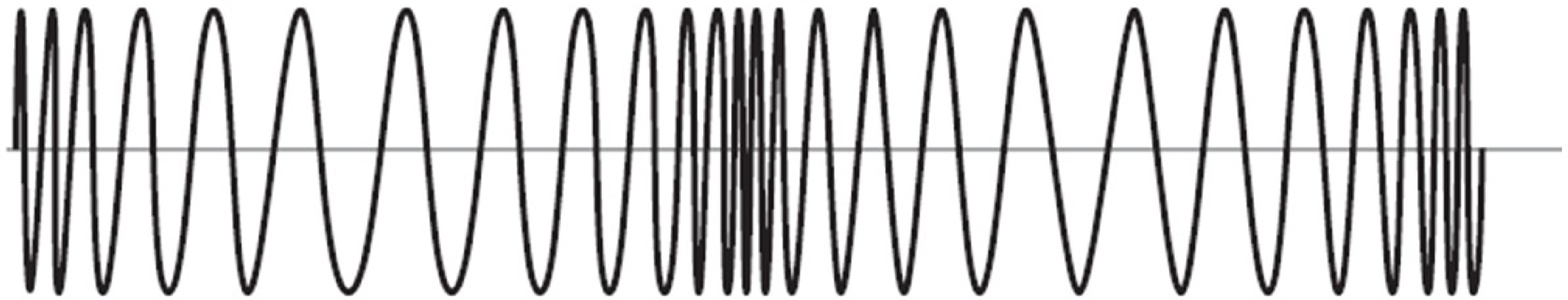
Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 3 Irregularity of zero-crossings

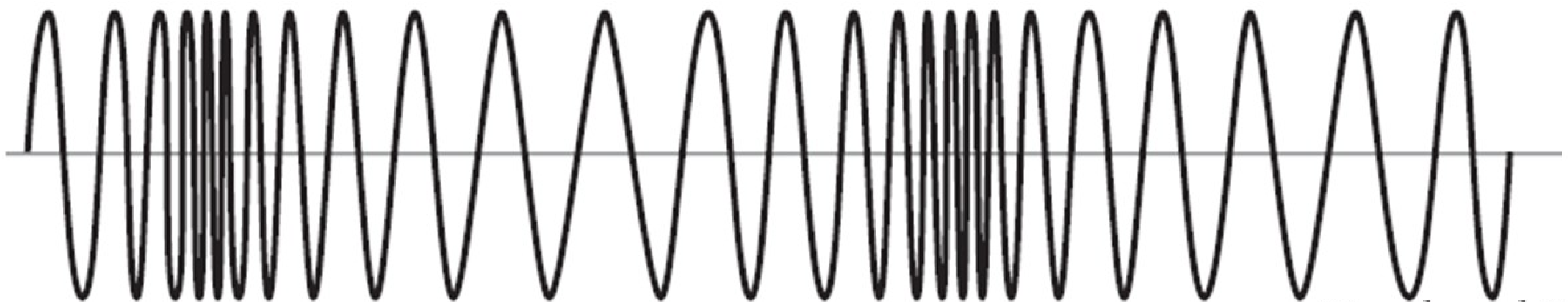
AM modulated Wave



PM modulated Wave



FM modulated Wave



Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 3 Irregularity of zero-crossings

We may cite two special cases where regularity is maintained in angle modulation:

1. The message signal m(t) increases or decreases linearly with time t, in which case the instantaneous frequency $\mathbf{f_i(t)}$ of the PM wave changes from the unmodulated carrier frequency $\mathbf{f_c}$ to a new constant value dependent on the slope of m(t)

Chapter 4.2: Properties of Angle-Modulated Waves

PROPERTY 3 Irregularity of zero-crossings

We may cite two special cases where regularity is maintained in angle modulation:

2. The message signal m(t) is maintained at some constant value, positive or negative, in which case the instantaneous frequency $\mathbf{f_i(t)}$ of the FM wave changes from the unmodulated carrier frequency $\mathbf{f_c}$ to a new constant value dependent on the constant value of m(t)

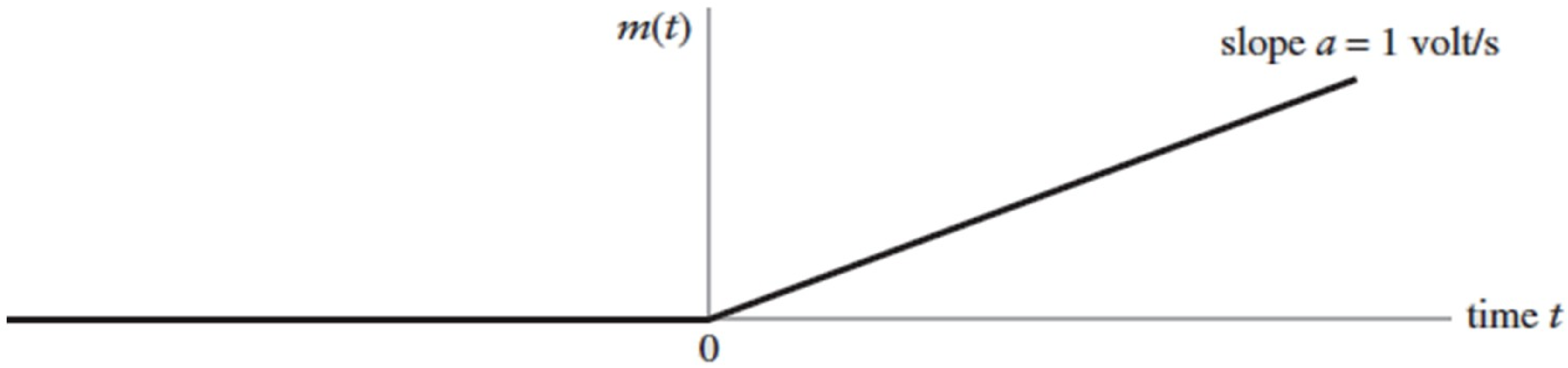
Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

- Consider a modulating wave m(t) that increases linearly with time t, starting at t=0, as shown by

$$m(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

a is the slope parameter (Figure 4.2a)



(a)

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

In what follows, we study the zero-crossings of the PM and FM waves produced by m(t) for the following set of parameters:

$$a = 1 \text{ volt/s} \quad f_c = \frac{1}{4} \text{Hz}$$

Phase Modulation:

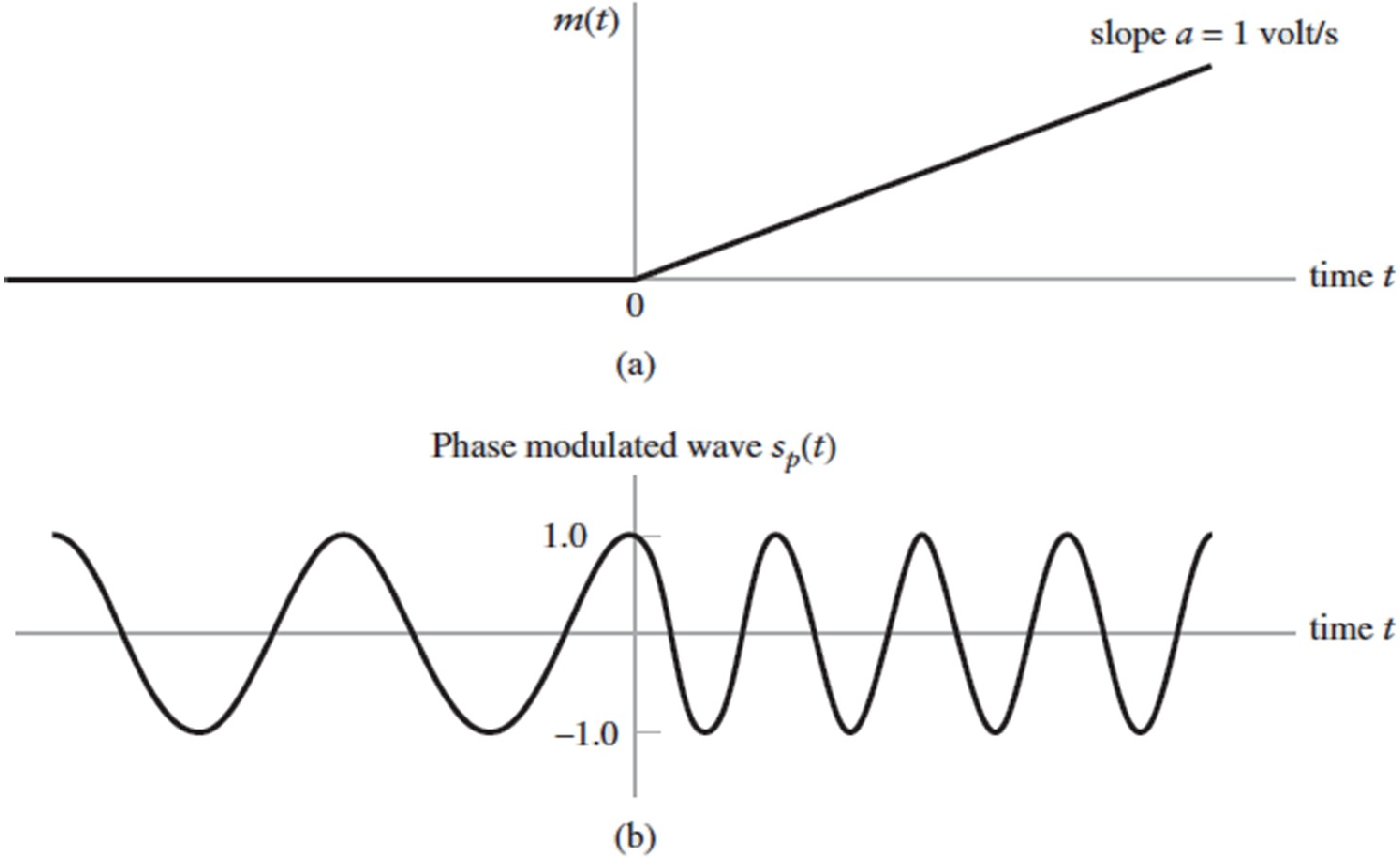
- Let phase-sensitivity factor $\mathbf{k_p} = \pi/2$ radians/volt. Applying Eq. (4.5) $s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \quad (4.5)$ to the given m(t) yields the PM wave

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

which is plotted in Fig. 4.2(b) for $A_c = 1$ volt.



Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

Let $\mathbf{t_n}$ denote the instant of time at which the PM wave experiences a zero crossing; this occurs whenever the **angle** of the PM wave is an odd multiple of $\pi/2$:

$$2\pi f_c t_n + k_p a t_n = \pi\left(2f_c + \frac{k_p a}{\pi}\right)t_n = \frac{\pi}{2} + n\pi, \quad n = 0, 1, 2, \ldots$$

$$t_n = \frac{\frac{1}{2} + n}{2f_c + \frac{k_p}{\pi}a} \quad \Rightarrow \quad t_n = \frac{1}{2} + n, \quad n = 0, 1, 2, \ldots$$

By substituting the given values for $\mathbf{f_c}$, **a** and $\mathbf{k_p}$ into this linear formula

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

Frequency Modulation:

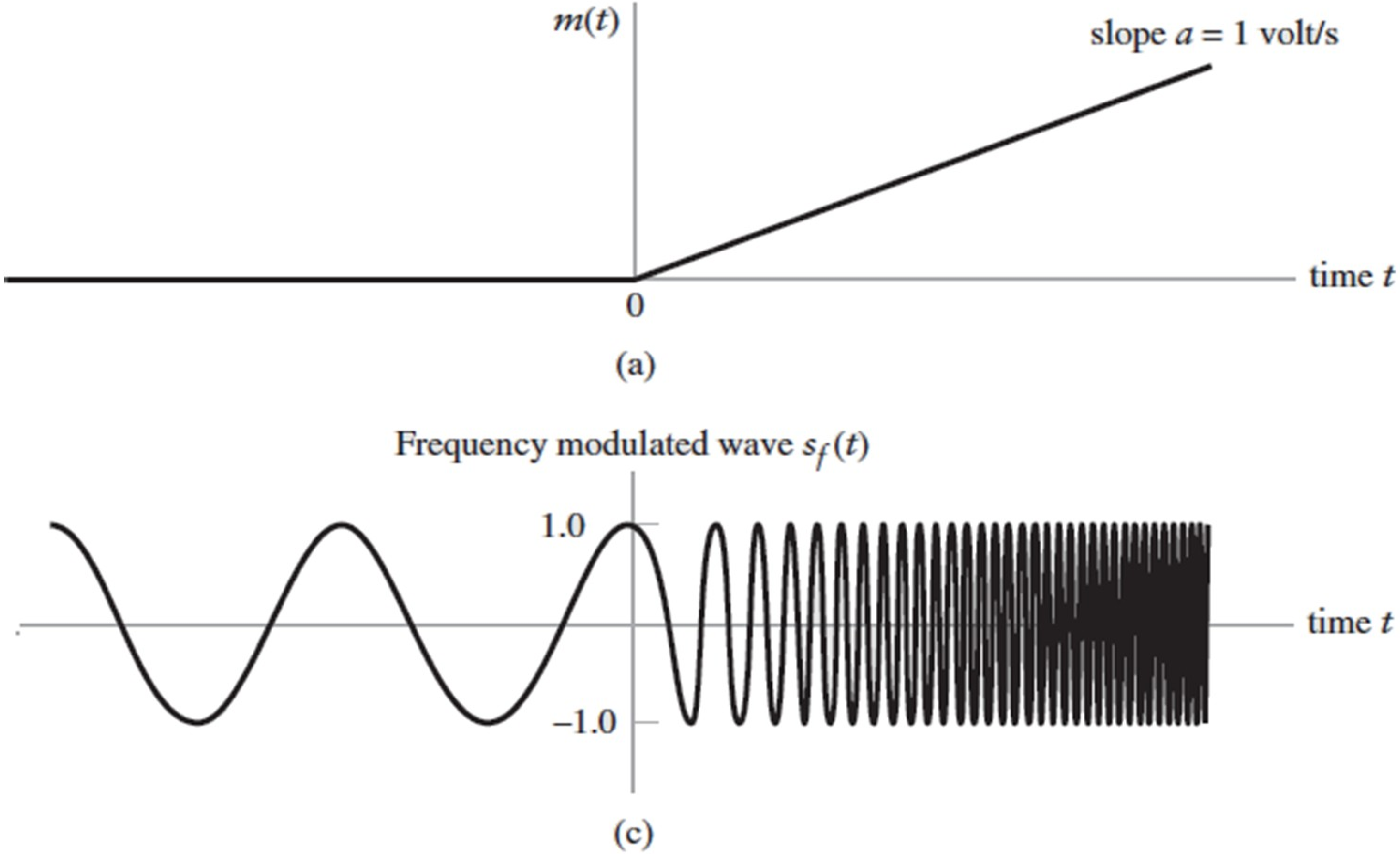
- Let frequency-sensitivity factor, $\mathbf{k_f}$ =1 Hz/volt. Applying Eq. (4.8) $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$ (4.8) yields the FM wave

$$s(t) = \begin{cases} A_c \cos\left(2\pi f_c t + \pi k_f a t^2\right), & t \geq 0 \\ A_c \cos\left(2\pi f_c t\right), & t < 0 \end{cases}$$

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

which is plotted in Figure 4.2*c*.



Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

Invoking the definition of a zero-crossing, we can obtain:

$$2\pi f_c t_n + \pi k_f a t_n^2 = \frac{\pi}{2} + n\pi, \;\; n = 0, 1, 2, \ldots$$

$$t_n = \frac{1}{a k_f}\left(-f_c + \sqrt{f_c^2 + a k_f\left(\frac{1}{2} + n\right)}\right), \;\; n = 0, 1, 2, \ldots$$

$$t_n = \frac{1}{4}\left(-1 + \sqrt{9 + 16n}\right), \;\; n = 0, 1, 2, \ldots$$

where $\mathbf{t}_n$ is again measured in seconds

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

Comparing the zero-crossing results derived for PM and FM waves, we may make the following observations once the linear modulating wave begins to act on the sinusoidal carrier wave:

1. For PM, regularity of the zero-crossings is maintained; the instantaneous frequency changes from the unmodulated value of $\mathbf{f_c = 1/4}$ Hz to the new constant value of $\mathbf{f_c + k_p}\,(\mathbf{a}/2\pi) = 0.5$Hz.

Recall that

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \qquad (4.2)$$

Chapter 4.2: Properties of Angle-Modulated Waves

Example 4.1 Zero-Crossings

2. For FM, the zero-crossings assume an irregular form; as expected, the instantaneous frequency increases linearly with time t.

Chapter 4.2: Properties of Angle-Modulated Waves

The angle-modulated waveforms of Fig. 4.2 should be contrasted with the corresponding ones of Fig. 4.1. Whereas in the case of sinusoidal modulation depicted in Fig. 4.1 it is difficult to discern the difference between PM and FM, this is not so in the case of Fig. 4.2.

In other words, depending on the modulating wave, it is possible for PM and FM to exhibit entirely different waveforms.

Chapter 4.2: Properties of Angle-Modulated Waves

Property 4 :Visualization difficulty of message waveform

The difficulty in visualizing the message waveform in angle-modulated waves is also attributed to the nonlinear character of angle-modulated waves.

Chapter 4.2: Properties of Angle-Modulated Waves

Property 4 :Visualization difficulty of message waveform

- In AM, we see the message waveform as the envelope of the modulated wave, provided the percentage modulation is less than 100 percent.

  (AM: The percentage modulation over 100 percent→phase reversal→distortion)

- This is not so in angle modulation, as illustrated by the corresponding waveform of Figures 4.1d and 4.1e for PM and FM, respectively.

Chapter 4.2: Properties of Angle-Modulated Waves

Property 5 :Tradeoff of increased transmission bandwidth for improved noise performance

- An important advantage of angle modulation over amplitude modulation is the realization of improved noise performance.
- This advantage is attributed to the fact that the transmission of a message signal by modulating the angle of a sinusoidal carrier wave is less sensitive to the presence of additive noise than transmission by modulating the amplitude of the carrier.

Chapter 4.2: Properties of Angle-Modulated Waves

Property 5 :Tradeoff of increased transmission bandwidth for improved noise performance

- The improvement in noise performance is achieved at the expense of a corresponding increase in the transmission bandwidth of angle modulation requirement modulation.
- Such a trade-off is not possible with amplitude modulation since the transmission bandwidth of an amplitude-modulated wave is fixed somewhere between the message bandwidth W and 2W, depending on the type of modulation employed

Section 4.3: Relationship Between PM and FM Waves

Section 4.3: Relationship Between PM and FM Waves

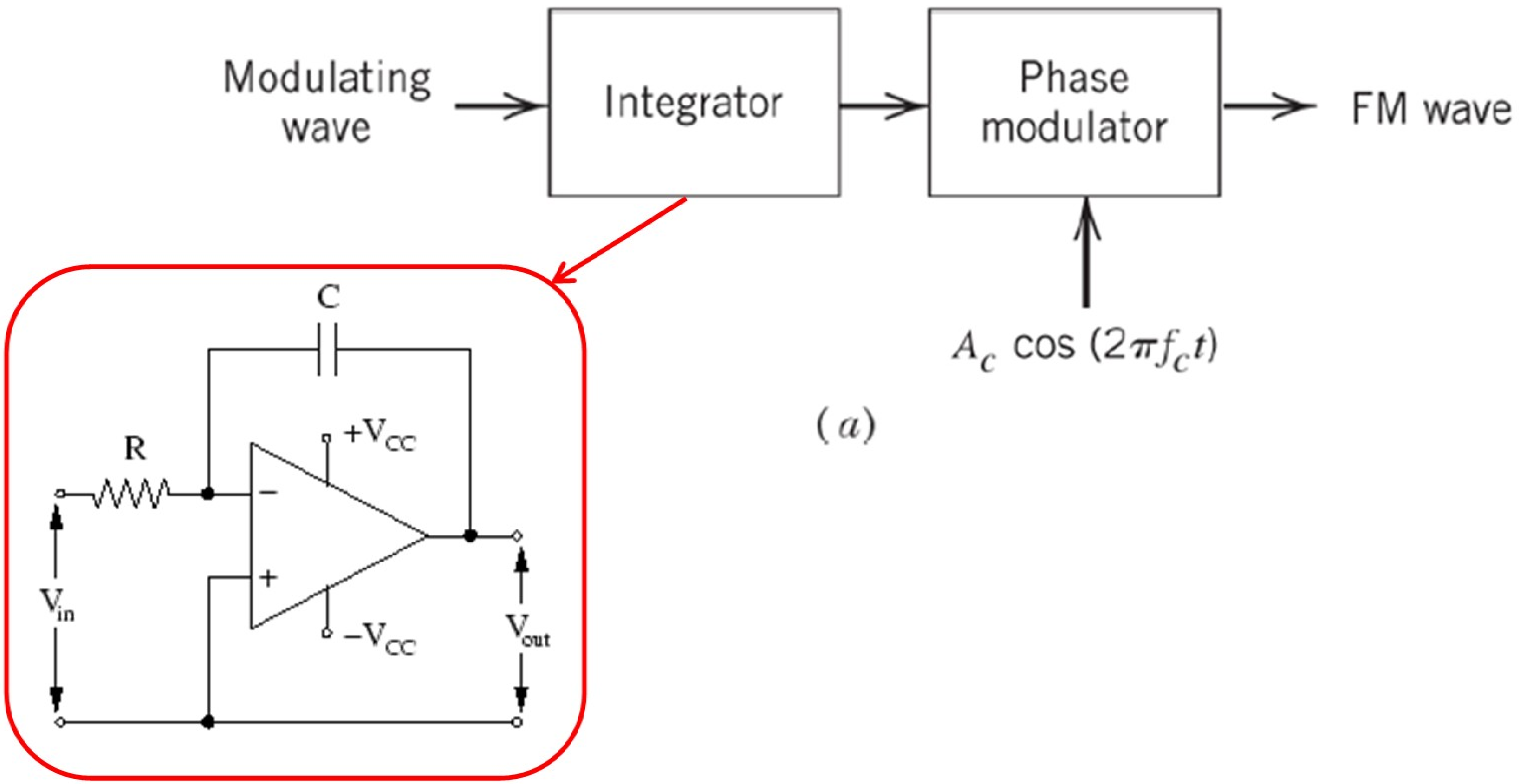
- Comparing Eq. (4.5) with (4.8) reveals that an FM signal may be regarded as a PM signal in which the modulating wave $\int_0^t m(\tau)d\tau$ is in place of m(t)

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \quad (4.5)$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \quad (4.8)$$

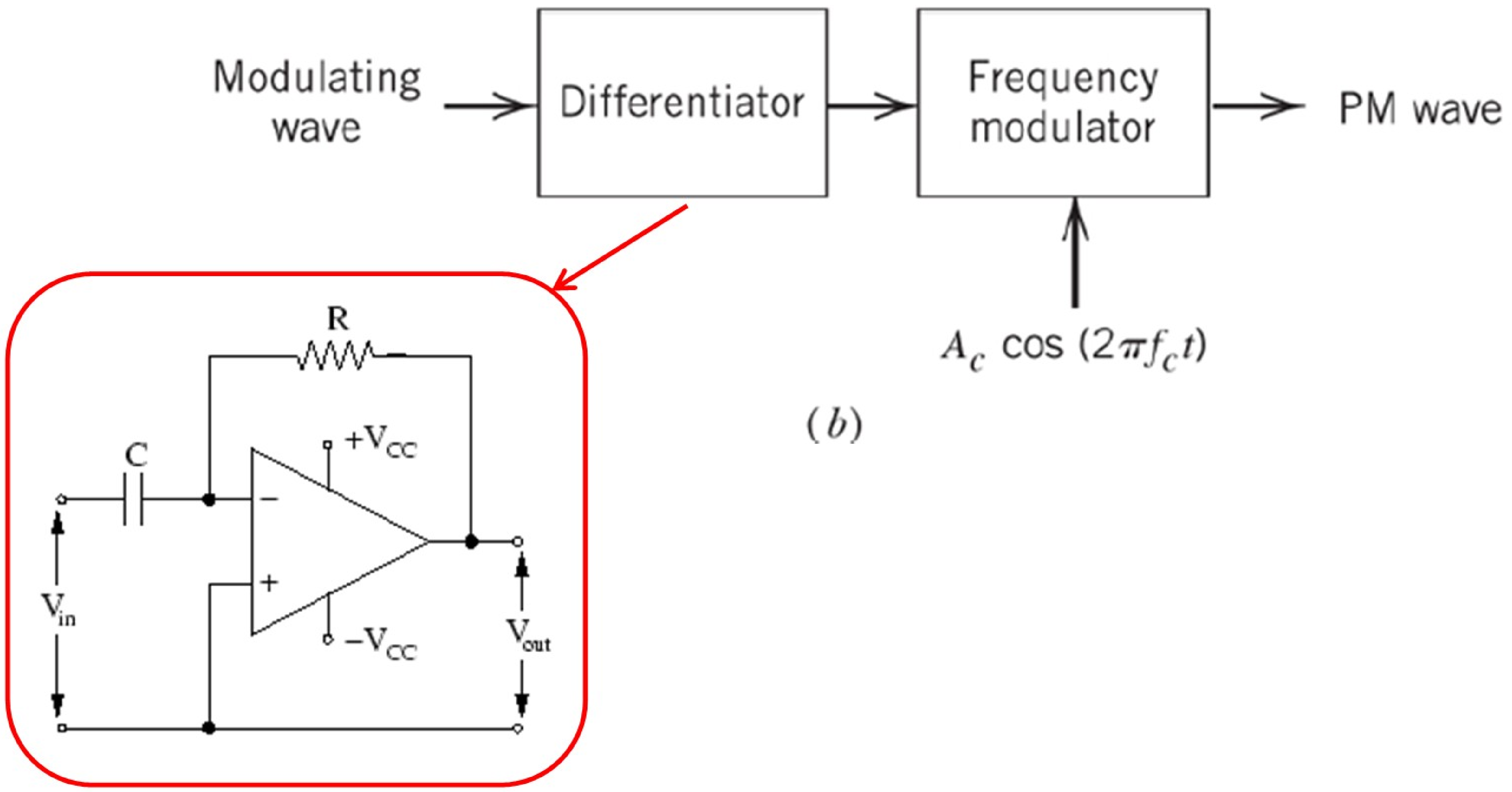
Section 4.3: Relationship Between PM and FM Waves

- The FM signal can be generated by first integrating m(t) and then using the result as the input to a phase modulator, as in Figure 4.3a



Section 4.3: Relationship Between PM and FM Waves

- Conversely, a PM signal can be generated by first differentiating m(t) and then using the result as the input to a frequency modulator, as in Figure 4.3b



(b)

Section 4.3: Relationship Between PM and FM Waves

- We may thus deduce all the properties of PM signals from those of FM signals and vice versa. Henceforth, we concentrate attention on FM signals.

Section 4.3: Relationship Between PM and FM Waves

- Summary

	$\theta_i(t)$	$f_i(t)$
Unmodulated signal	$2\pi f_c t$	f_c
PM signal	$2\pi f_c t + k_p m(t)$	$f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$
FM signal	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau$	$f_c + k_f m(t)$

Section 4.4: Frequency Modulation

Section 4.4: Frequency Modulation

- The FM signal s(t) defined by Eq. (4.8) is a nonlinear function of the modulating signal m(t), which makes FM a nonlinear modulation process $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right] \quad (4.8)$
- How then can we tackle the spectral analysis of FM signal?

We propose to provide an empirical answer to this important question by proceeding in the same manner as with AM modulation, that is,

1. We first consider the simple case of a single-tone modulation that produces a narrowband FM wave (Narrow bandwidth)
2. We next consider the more general case also involving a single-tone modulation, but this time the FM wave is wide-band

Section 4.4: Frequency Modulation

Our immediate objective is to establish an empirical relationship between ***the transmission bandwidth*** of an FM wave and the ***message bandwidth***

Consider then a sinusoidal modulating signal define by

$$m(t) = A_m \cos(2\pi f_m t)$$

The instantaneous frequency of the resulting FM signal is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t) \qquad (4.11)$$

$$\Delta f = k_f A_m \qquad (4.12)$$

Section 4.4: Frequency Modulation

The quantity **Δf** is called the ***frequency deviation***, representing the maximum departure of the instantaneous frequency of the FM signal from the carrier frequency $\mathbf{f_c}$

A fundamental characteristic of an FM signal is that the frequency deviation Δf is proportional to the amplitude of the modulating signal and is independent of the modulating frequency

Using Eq. (4.11), the angle θi(t) of the FM signal is obtained as

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\,dt = 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)$$

Section 4.4: Frequency Modulation

The ratio of the frequency deviation Δf to the modulation frequency f_m is commonly called the ***modulation index*** of the FM wave. We denote this new parameter by β so we write

$$\beta = \frac{\Delta f}{f_m} \qquad (4.13)$$

And

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \qquad (4.14)$$

Section 4.4: Frequency Modulation

The parameter **β** represents the ***angle deviation of the FM signal***, i.e. the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier. **β** is measured in **radians**.

The FM signal itself is given by

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \tag{4.16}$$

- Depending on the value of the modulation index β, we may distinguish two cases of frequency modulation:

◊ ***Narrow-band FM***, for which β is small compared to one radian

◊ ***Wide-band FM***, for which β is large compared to one radian

Section 4.4: Narrow-Band Frequency Modulation

Narrow-band frequency modulation

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \qquad (4.16)$$

- Consider Eq. (4.16), which defines an FM signals resulting from the use of sinusoidal modulating signal. Expanding this relation, we get

$$s(t) = A_c \cos(2\pi f_c t)\cos\left[\beta \sin(2\pi f_m t)\right] - A_c \sin(2\pi f_c t)\sin\left[\beta \sin(2\pi f_m t)\right] \quad (4.17)$$

- Assuming that the modulation index **β** is small compared to one radian, we may use the following two approximations:

$$\cos\left[\beta \sin(2\pi f_m t)\right] \simeq 1 \qquad \sin\left[\beta \sin(2\pi f_m t)\right] \simeq \beta \sin(2\pi f_m t)$$

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (4.18)$$

Section 4.4: Narrow-Band Frequency Modulation

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (4.18)$$

$$\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$s(t) \simeq A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c \left\{\cos\left[2\pi(f_c+f_m)t\right] - \cos\left[2\pi(f_c-f_m)t\right]\right\} \quad (4.19)$$

- This expression is somewhat similar to the corresponding one defining an AM signal (from Example 3.1):

$$s_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \left\{\cos\left[2\pi(f_c+f_m)t\right] + \cos\left[2\pi(f_c-f_m)t\right]\right\} \quad (4.20)$$

Section 4.4: Narrow-Band Frequency Modulation

- Compare Eqs. (4.19) and (4.20), we see that the basic difference between an AM signal and a narrow-band FM signal is that the algebraic sign of the lower side frequency in the narrow-band FM is reversed

➢ ***Thus, a narrow-band FM signal requires essentially the same AM signal transmission bandwidth (i.e. $2f_m$) as AM signal***

Section 4.4: Narrow-Band Frequency Modulation

Equation (4.18) defines the approximate form of a narrow-band FM wave produced by the sinusoidal modulating wave $A_m \cos(2\pi f_m t)$

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

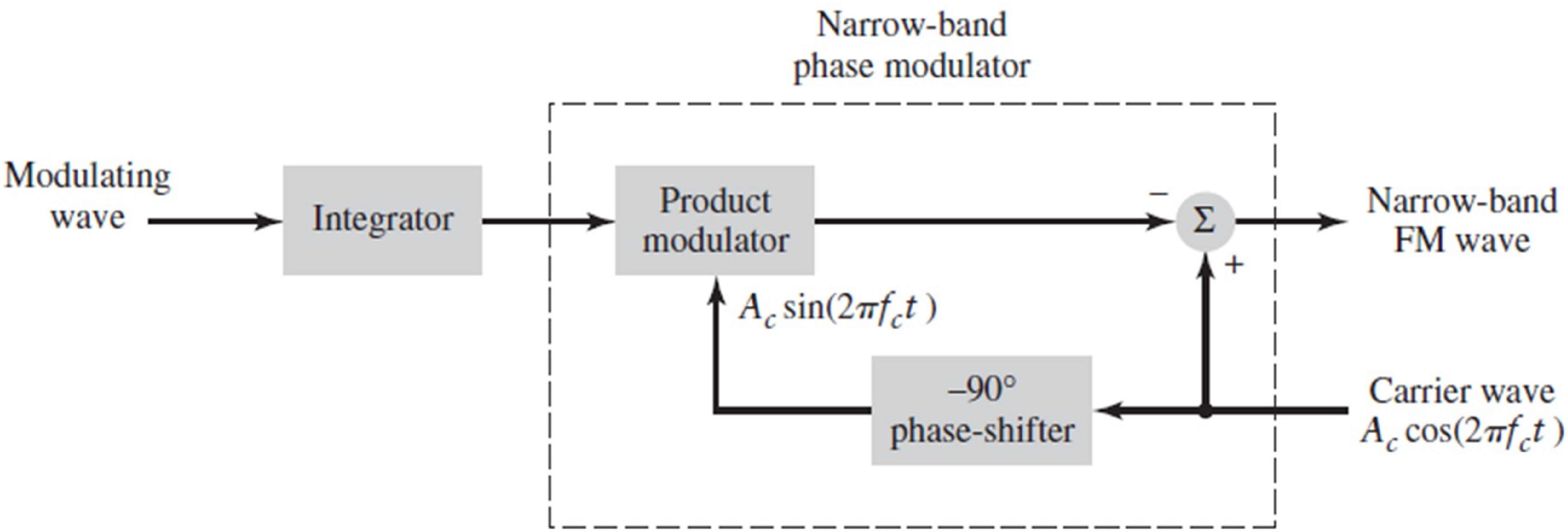


FIGURE 4.4 Block diagram of an indirect method for generating a narrow-band FM wave.

Section 4.4: Narrow-Band Frequency Modulation

Phase Noise

- *Phase noise* is often introduced by oscillators in band-pass communications and has a number of causes.
- Some causes are the ***deterministic***, such as those created by changes in oscillator ***temperature***, ***supply voltage***, ***physical vibration***, ***magnetic field***, ***humidity***, or ***output load*** impedance.
- The phase noise due to these sources may be minimized ***by good design.***

Section 4.4: Narrow-Band Frequency Modulation

- ◊ Other sources are categorized as ***random***, which can be controlled but not eliminated by appropriate circuitry, such as ***phase-lock loops (PLL).***
- ◊ The ***phase noise*** introduced by oscillators has a multiplicative effect on an angle-modulated signal.

Section 4.5: Wide-Band Frequency Modulation

Wide-band frequency modulation

◊ The following studies the spectrum of the single-tone FM signal of Eq. (4.16) for an arbitrary value of the modulation index β.

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \quad (4.16)$$

◊ By using the complex representation of band-pass signals described in Chapter 2: (Carrier frequency $\mathbf{f_c}$ compared to the bandwidth of the FM signal is large enough)

$$s(t) = \text{Re}\left[A_c \exp\left(j2\pi f_c t + j\beta \sin(2\pi f_m t)\right)\right] \quad (4.21)$$

$$= \text{Re}\left[\tilde{s}(t)\exp(j2\pi f_c t)\right]$$

where $\tilde{s}(t) = A_c \exp\left[j\beta \sin(2\pi f_m t)\right] \rightarrow$ periodic function

Section 4.5: Wide-Band Frequency Modulation

$$\tilde{s}(t) = A_c \exp\left[j\beta \sin(2\pi f_m t) \right]$$

◊ We may therefore expend $\tilde{s}(t)$ in the form of complex Fourier series as follows:

$$\tilde{s}(t) = \sum_{n=-\infty} c_n \exp(j2\pi n f_m t) \qquad (4.23)$$

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt$$

$$= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp\left[j\beta \sin(2\pi f_m t) - j2\pi n f_m t \right] dt \qquad (4.24)$$

$x = 2\pi f_m t$

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp\left[j(\beta \sin x - nx) \right] dx \qquad (4.26)$$

$$c_n = A_c J_n(\beta) \quad \because J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left[j(\beta \sin x - nx) \right] dx \qquad (4.28)$$

*n*th order Bessel function of the first kind.

$$s(t) = A_c \cdot \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left[j2\pi (f_c + n f_m) t \right] \right] \qquad (4.31)$$

Section 4.5: Wide-Band Frequency Modulation

◊ Taking the Fourier transforms of both sides of Eq. (4.31)

$$S(f)=\frac{A_c}{2}\sum_{n=-\infty}^{\infty}J_n(\beta)\left[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)\right] \quad (4.32)$$

◊ In Figure 4.6 we have plotted the Bessel function $\mathbf{J_n(\beta)}$ versus the modulation index $\boldsymbol{\beta}$ for different positive integer values of **n**.

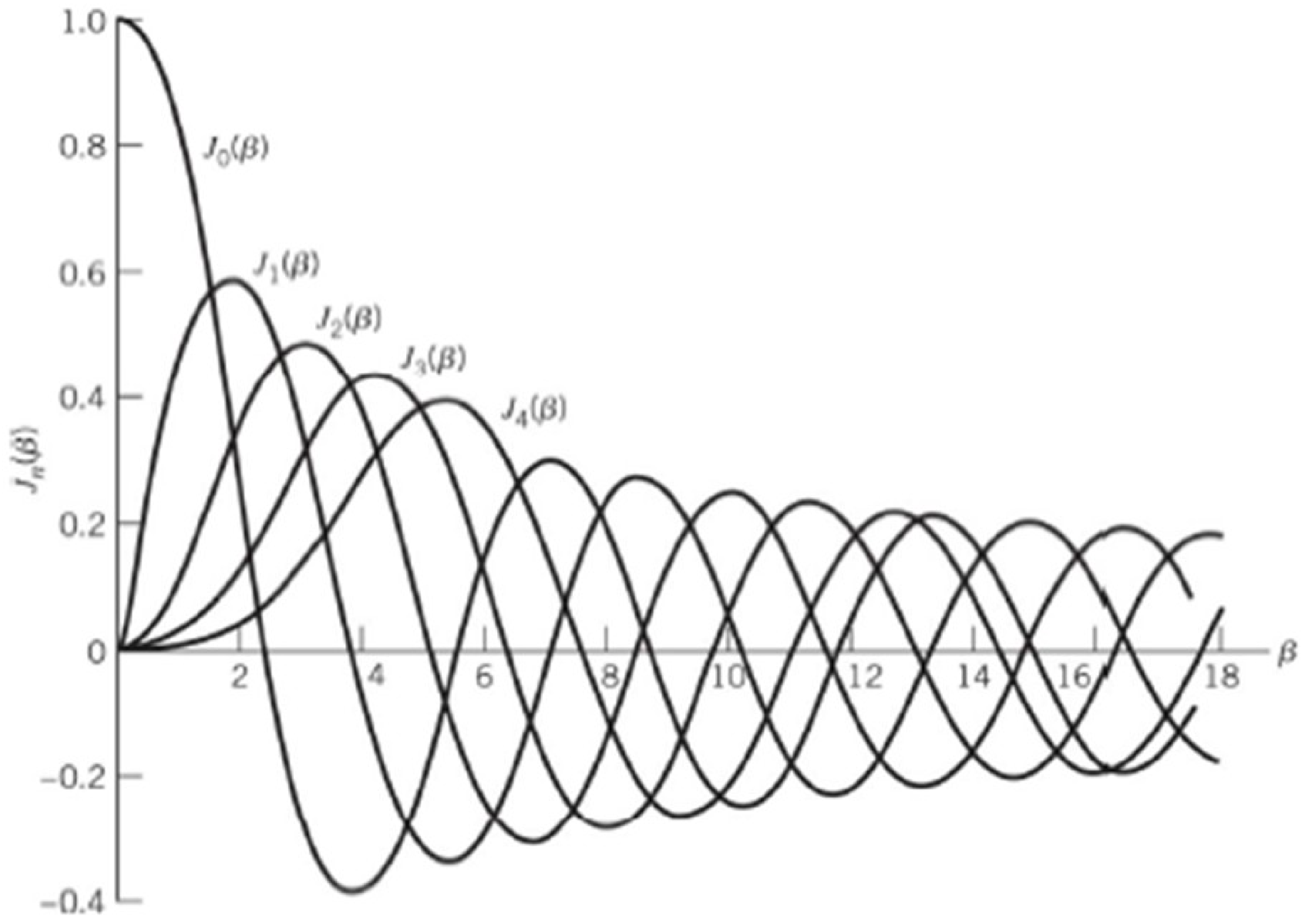


FIGURE4.6 Plots of Bessel functions of the first kind.

Section 4.5: Wide-Band Frequency Modulation

◊ We can develop further insight into the behavior of the Bessel function $\mathbf{J_n(\beta)}$ by ***making use of the following properties***:

1. For **n** *even*, we have $\mathbf{J_n(\beta)=J_{-n}(\beta)}$; on the other hand, for **n** *odd*, we have $\mathbf{J_n(\beta) = -J_{-n}(\beta)}$. That is

$$J_n(\beta)=(-1)^n J_{-n}(\beta) \quad \text{for all } n \qquad (4.33)$$

2. For ***small values of the modulation index β***, we have

$$\left.\begin{aligned} J_0(\beta) &\simeq 1 \\ J_1(\beta) &\simeq \frac{\beta}{2} \\ J_n(\beta) &\simeq 0, \quad n>2 \end{aligned}\right\} \qquad (4.34)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta)=1 \qquad (4.35)$$

Section 4.5: Wide-Band Frequency Modulation

◊ Thus, using Eqs. (4.32) through (4.35) and the curves of Figure 4.6, ***we may make the following observations:***

1. The spectrum of an FM signal contains a carrier component **(n=0)** and *an infinite set of side frequencies* located symmetrically on either side of the carrier at frequency separations of $\mathbf{f_m}$, $\mathbf{2f_m}$, $\mathbf{3f_m}$, ….
2. (An AM system gives rise to only one pair of side frequencies.)

Section 4.5: Wide-Band Frequency Modulation

2. For the special case of β small compared with unity, only the Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values (see 4.34), so that the FM signal is effectively composed of a carrier and ***a single pair of side frequencies at*** $f_c \pm f_m$.

(This situation corresponds to the special case of narrowband FM that was considered previously)

Section 4.5: Wide-Band Frequency Modulation

3. The envelope of an FM signal is constant, so that the average power of such a signal developed across a 1–ohm resistor is also constant, as shown by

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$P = \frac{1}{2} A_c^2 \quad \text{(Using (4.31) and (4.35))} \qquad (4.36)$$

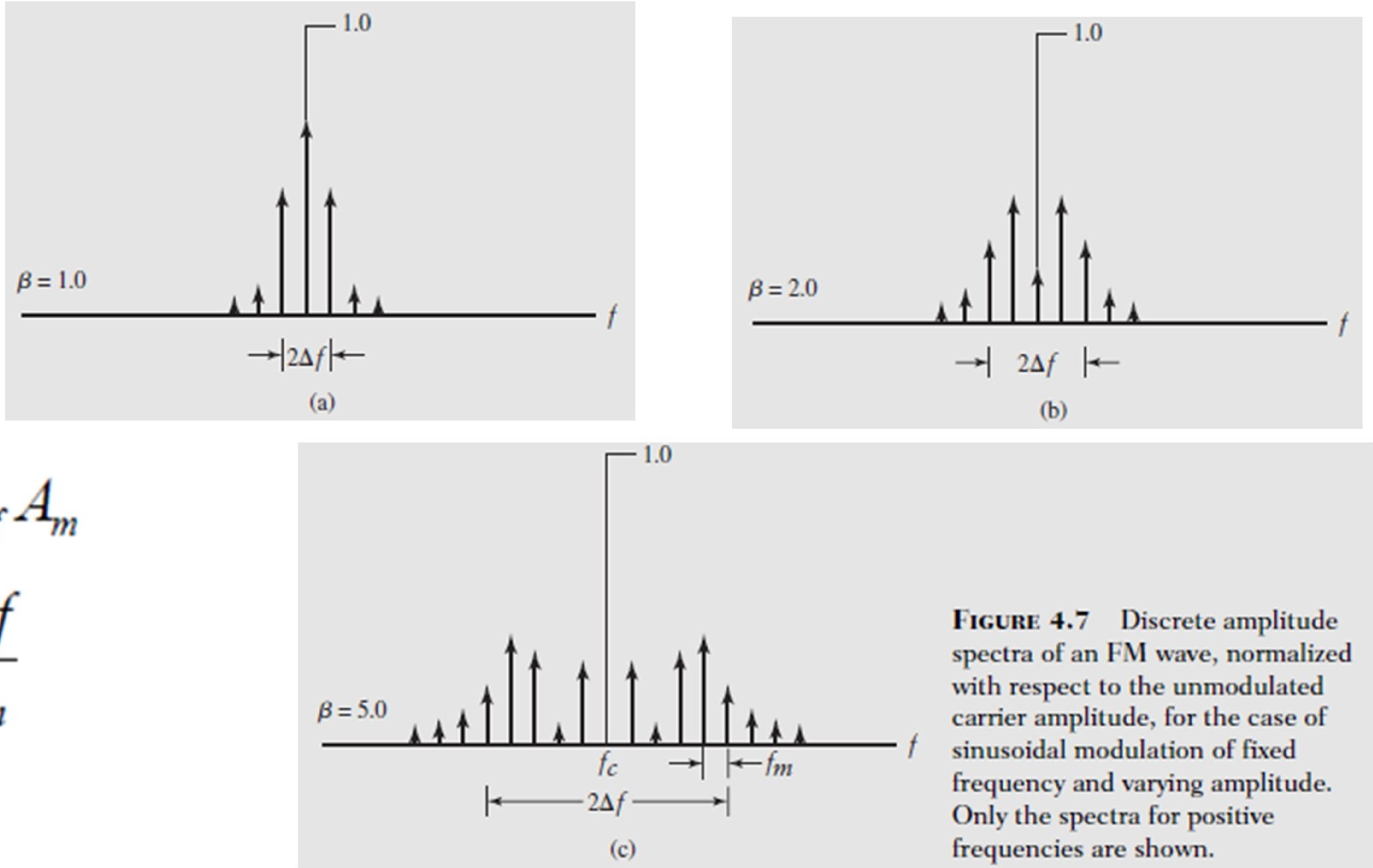
Section 4.5: Wide-Band Frequency Modulation

EXAMPLE 4.3: Spectra of FM Signals

- ◊ In this example, we wish to investigate the ways in which variations in the amplitude and frequency of a sinusoidal modulating signal affect the spectrum of the FM signal.
- ◊ Consider first the case when the frequency of the modulating signal is fixed, but its amplitude is varied, producing a corresponding variation in the frequency deviation Δf.

Section 4.5: Wide-Band Frequency Modulation

EXAMPLE 4.3: Spectra of FM Signals



$$\Delta f = k_f A_m$$

$$\beta = \frac{\Delta f}{f_m}$$

FIGURE 4.7 Discrete amplitude spectra of an FM wave, normalized with respect to the unmodulated carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown.

Section 4.5: Wide-Band Frequency Modulation

EXAMPLE 4.3: Spectra of FM Signals

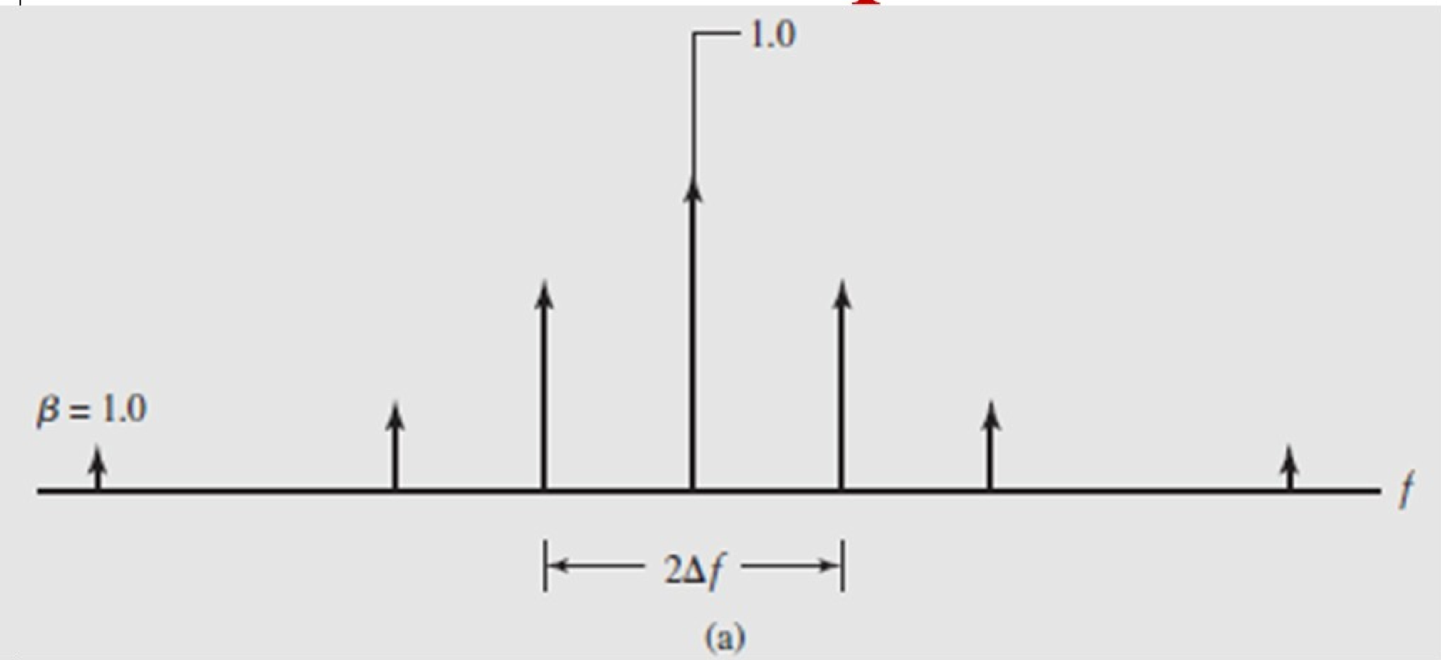
◊ Consider next the case when the amplitude of the modulating signal is fixed; that is, the frequency deviation Δf is maintained constant, and the modulation frequency f_m is varied.

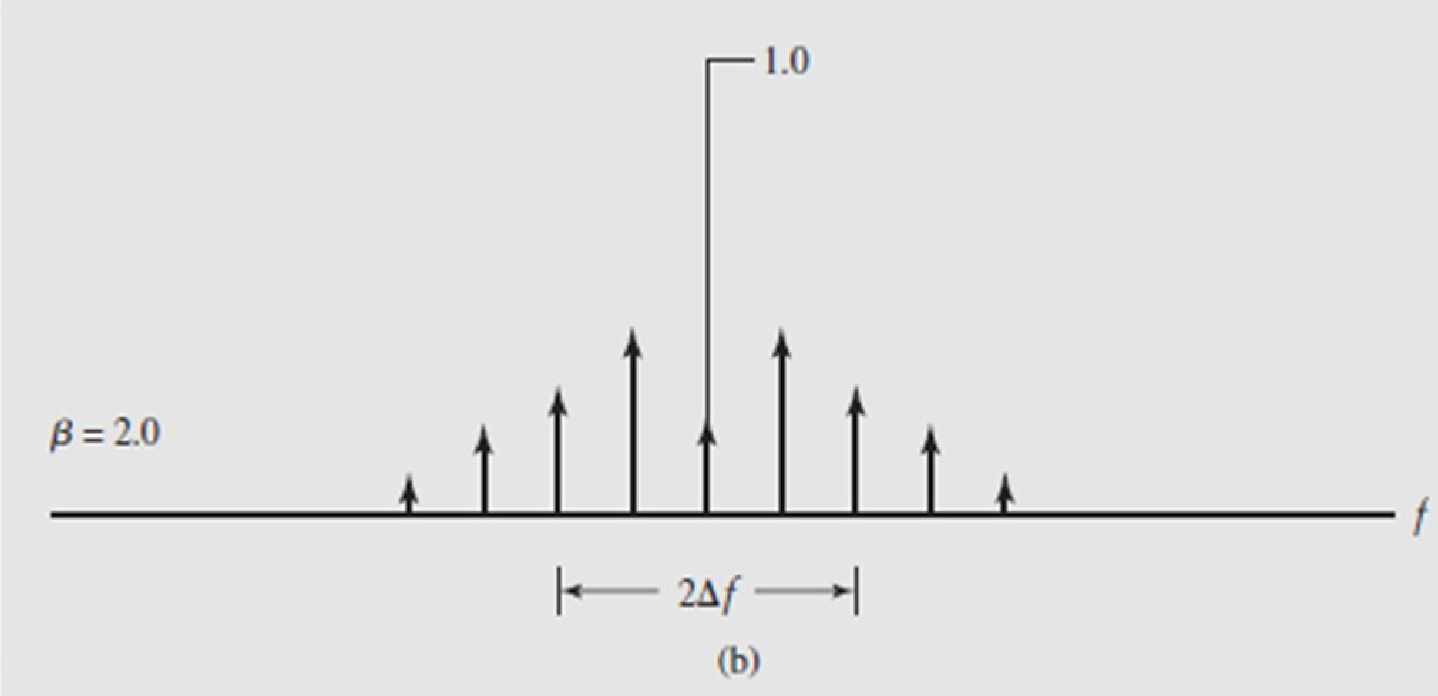
◊ We have an increasing number of spectral lines crowding into the fixed frequency interval $f_c - \Delta f < | f | < f_c + \Delta f$.

◊ When **β** approaches infinity, the bandwidth of the FM wave approaches the limiting value of **2Δf**, which is an important point to keep in mind.

Section 4.5: Wide-Band Frequency Modulation

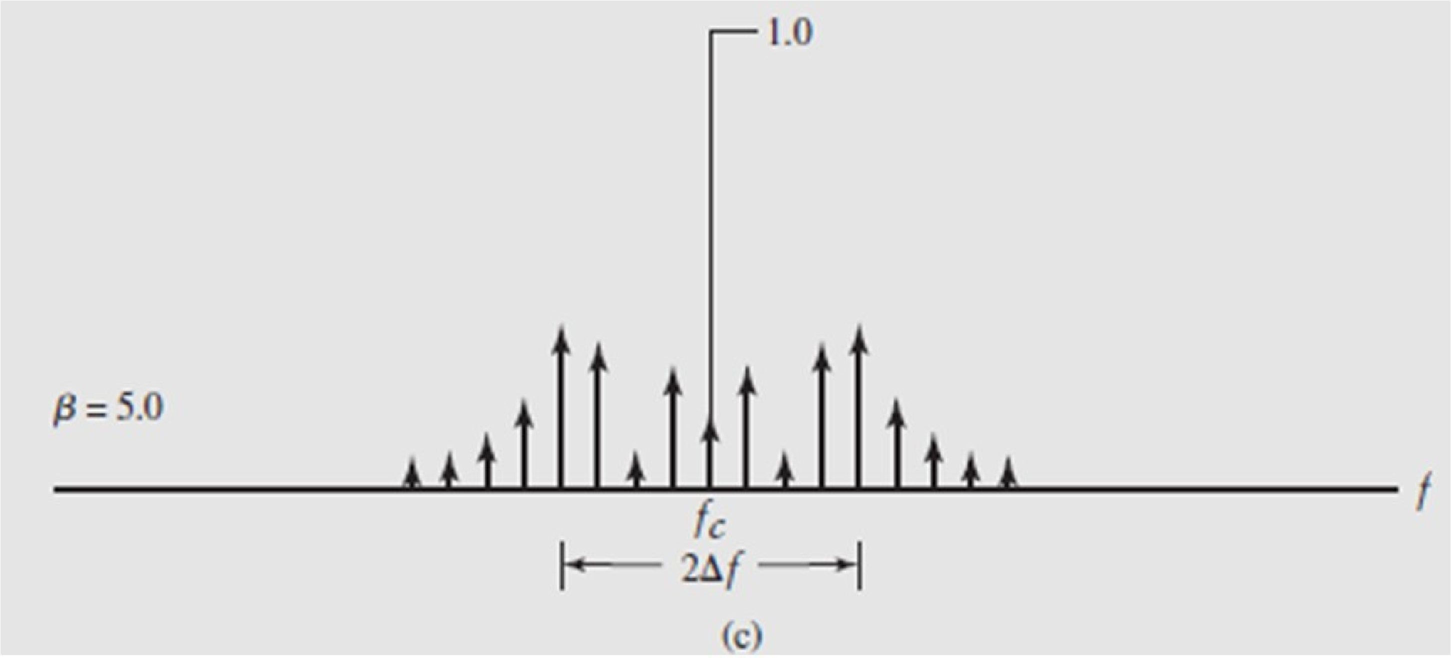
EXAMPLE 4.3: Spectra of FM Signals





$$\Delta f = k_f A_m$$

$$\beta = \frac{\Delta f}{f_m}$$



Section 4.6: Transmission Bandwidth of FM Signals

Transmission Bandwidth of FM Signals

- ◊ In theory, an FM signal contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal is similarly infinite in extent.
- ◊ In practice, however, we find that the FM signal is effectively limited to a finite number of significant side frequencies compatible with a specified amount of distortion.

Section 4.6: Transmission Bandwidth of FM Signals

Transmission Bandwidth of FM Signals

◊ Consider the case of an FM signal generated by a single-tone modulating wave of frequency $\mathbf{f_m}$.

- In such an FM signal, the side frequencies that are separated from the carrier frequency $\mathbf{f_c}$ by an amount greater than the frequency deviation $\mathbf{\Delta f}$ ***decrease*** rapidly toward zero, so that the bandwidth always exceeds the total frequency excursion, but nevertheless is limited.

Section 4.6: Transmission Bandwidth of FM Signals

Transmission Bandwidth of FM Signals

◊ We may thus define an approximate rule for the transmission bandwidth of an FM signal generated by a single-tone modulating signal of frequency $\mathbf{f_m}$ as follows:

$$B_T \simeq 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right) \qquad (4.38)$$

Large $\beta \rightarrow B_T \simeq 2\Delta f$

Small $\beta \rightarrow B_T \simeq 2f_m$

This empirical relation is known as **Carson's rule**.

Section 4.6: Transmission Bandwidth of FM Signals

Transmission Bandwidth of FM Signals

◊ For a more accurate assessment of the bandwidth requirement of an FM signal, we may thus define the transmission bandwidth of an FM wave as ***the separation between the two frequencies beyond which none of the side frequencies is greater than 1% of the carrier amplitude*** obtained when the modulation is removed.

◊ That is, we define the transmission bandwidth as $2n_{max}f_m$, where f_m is the modulation frequency and n_{max} is the largest value of the integer $\mathbf{n}$ that satisfies the requirement $|J_n(\beta)| > 0.01$

Section 4.6: Transmission Bandwidth of FM Signals

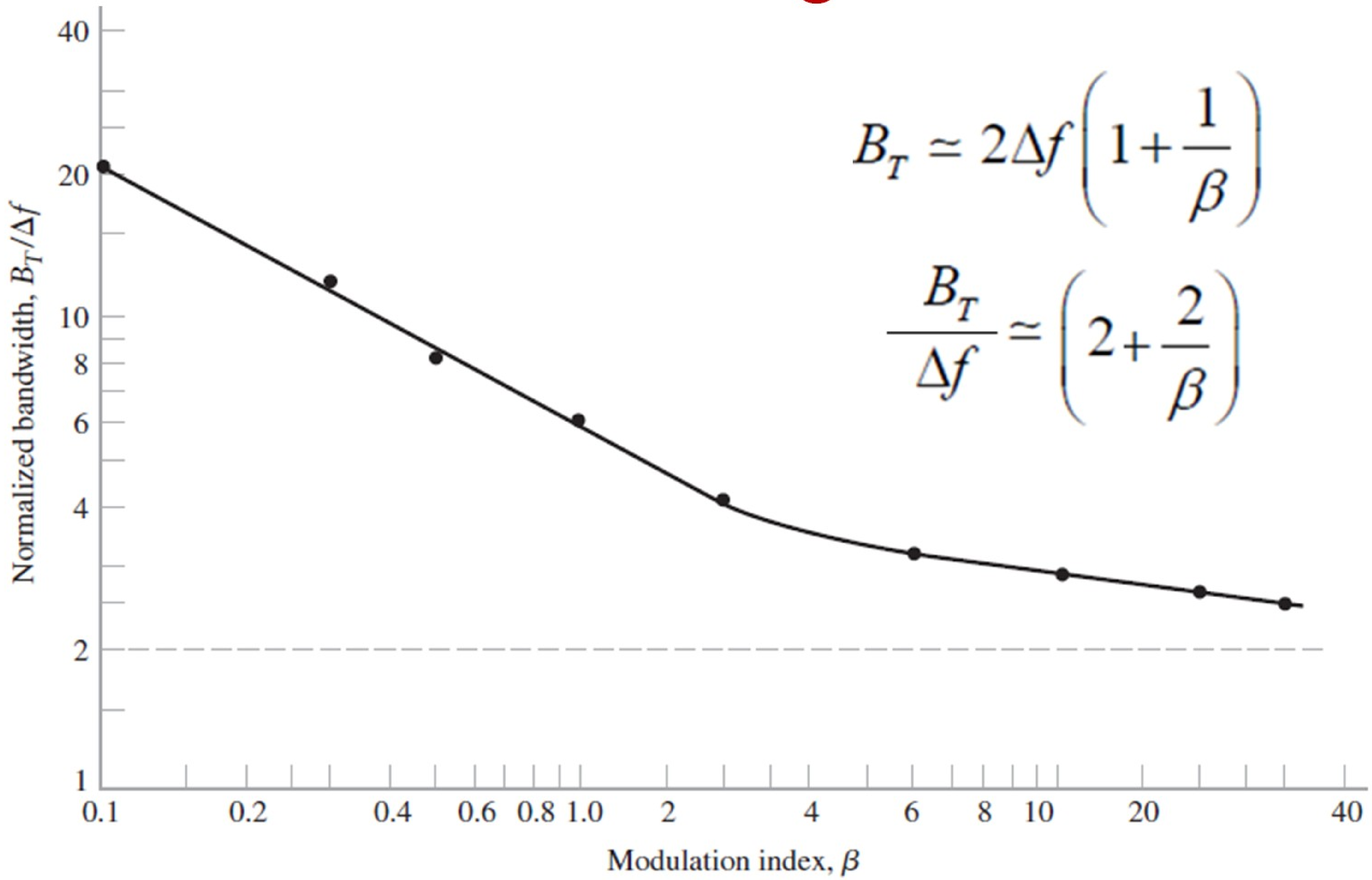
Transmission Bandwidth of FM Signals

TABLE 4.2 ***Number of Significant Side-Frequencies of a Wide-Band FM Signal for Varying Modulation Index***

Modulation Index β	*Number of Significant Side-Frequencies* $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

Section 4.6: Transmission Bandwidth of FM Signals

Transmission Bandwidth of FM Signals



$$B_T \simeq 2\Delta f\left(1+\frac{1}{\beta}\right)$$

$$\frac{B_T}{\Delta f} \simeq \left(2+\frac{2}{\beta}\right)$$

FIGURE 4.9 Universal curve for evaluating the one percent bandwidth of an FM wave.

Section 4.6: Transmission Bandwidth of FM Signals

ARBITRARY MODULATING WAVE

- Consider an arbitrary modulating wave m(t) with W denotes the message bandwidth
- One way of tackling it is to seek a ***worst-case evaluation*** of the transmission bandwidth
- We first determine the so-called ***deviation ratio*** **D**, defined as the ratio of the frequency deviation **Δf**, which corresponds to the maximum possible amplitude of the modulation wave m(t) to the highest modulation frequency *W*

$$D = \frac{\Delta f}{W}$$

Section 4.6: Transmission Bandwidth of FM Signals

ARBITRARY MODULATING WAVE

- The deviation ratio **D** plays the same role for nonsinusoidal modulation that the modulation index **β** plays for the case of sinusoidal modulation, so

$$B_T = 2(\Delta f + W)$$

This is the *generalized Carson rule* for the transmission bandwidth of an arbitrary FM signal

- From a practical viewpoint, the generalized Carson rule somewhat ***underestimates*** the bandwidth requirement of an FM system, whereas, in a corresponding way, using the universal curve yields a somewhat conservative result

Section 4.6: Transmission Bandwidth of FM Signals

Example

- In North America, the maximum value of frequency deviation is fixed at 75 kHz for commercial FM broadcasting by radio. If we take the modulation frequency ***W =15 kHz***, which is typically the "maximum" audio frequency of interest in FM transmission, we find that the corresponding value of the deviation ratio is [using Eq. (4.38)]

$$D = \frac{75}{15} = 5$$

Section 4.6: Transmission Bandwidth of FM Signals

Example

- Using the values $\Delta f = 75\ kHz$ and $D = 5$, in the generalized Carson rule of Eq. (4.39), we find that the approximate value of the transmission bandwidth of the FM signal is obtained as

$$B_T = 2(75 + 15) = 180\ \text{kHz}$$

- On the other hand, use of the universal curve of Fig. 4.9 gives the transmission bandwidth of the FM signal to be

$$B_T = 3.2\ \Delta f = 3.2 \times 75 = 240\ \text{kHz}$$

- In this example, ***Carson's rule underestimates the transmission bandwidth by 25 percent*** compared with the result of using the universal curve of Fig. 4.9

Section 4.7: Generation of FM Waves

Generation of FM Waves

- According to Eq. (4.5), the instantaneous frequency $\mathbf{f_i(t)}$ of an FM wave varies linearly with the message signal m(t).

$$f_i(t) = f_c + k_f m(t) \qquad (4.5)$$

- For the design of a ***frequency modulator***, we therefore need a device that produces an output signal whose instantaneous frequency is sensitive to variations in the amplitude of an input signal in a linear manner

Section 4.7: Generation of FM Waves

There are two basic methods of generating frequency-modulated waves, one direct and the other indirect

DIRECT METHOD

- The direct method uses a sinusoidal oscillator, with one of the reactive elements (e.g., capacitive element) in the tank circuit of the oscillator being directly controllable by the message signal

Disadvantage:

A serious limitation of the direct method is the ***tendency for the carrier frequency to drift***, which is usually unacceptable for commercial radio applications

Section 4.7: Generation of FM Waves

DIRECT METHOD

- To overcome this limitation, frequency stabilization of the FM generator is required, which is realized ***through the use of feedback around the oscillator***
- Although the oscillator may itself be simple to build, the use of frequency stabilization adds system complexity to the design of the frequency modulator

Section 4.7: Generation of FM Waves

INDIRECT METHOD: ARMSTRONG MODULATOR

- The message signal is first used to produce a narrow-band FM, which is followed by frequency multiplication to increase the frequency deviation to the desired level
- This modulation scheme is called the Armstrong wide-band frequency modulator

Section 4.7: Generation of FM Waves

INDIRECT METHOD: ARMSTRONG MODULATOR

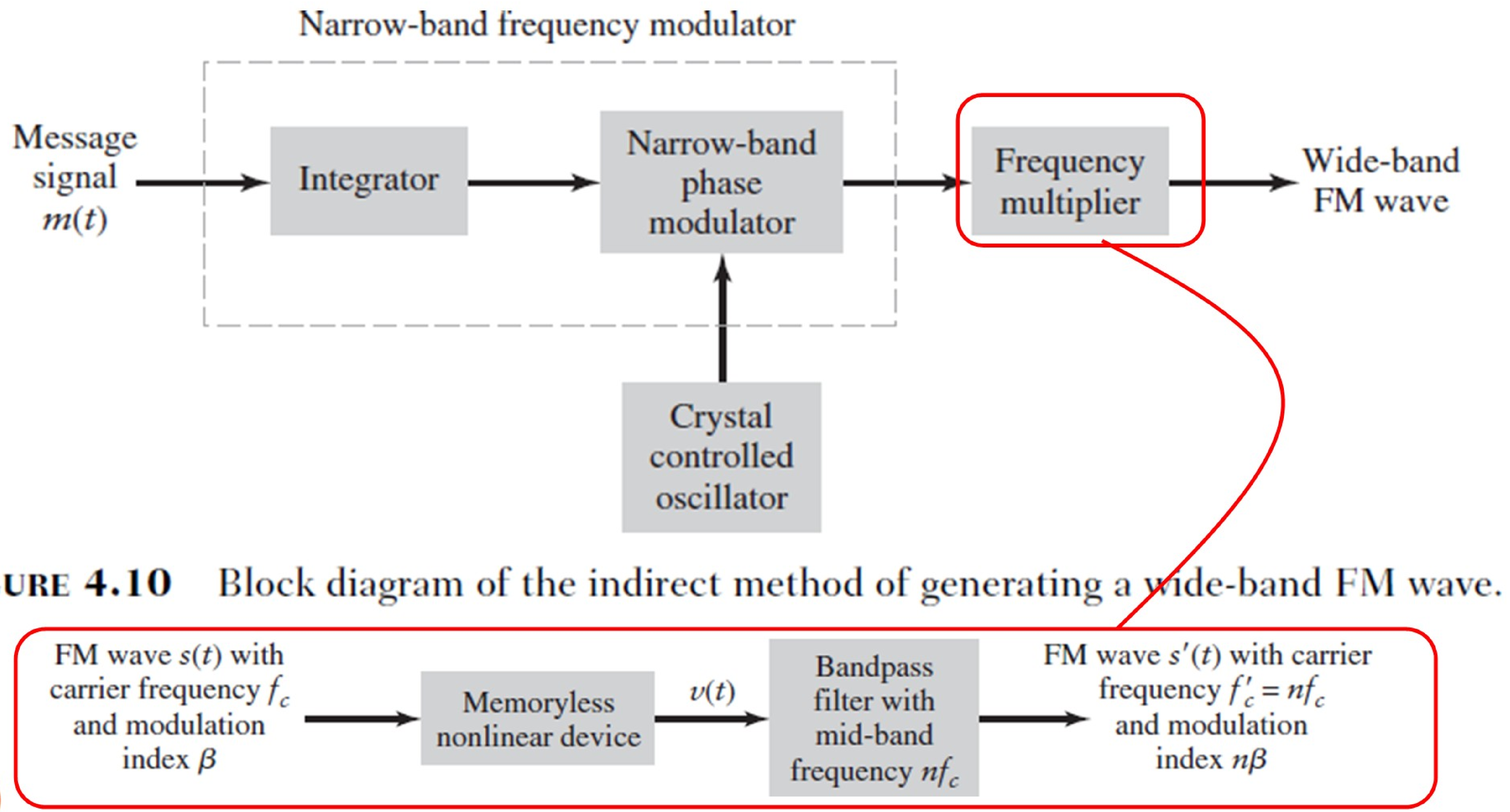


FIGURE 4.10 Block diagram of the indirect method of generating a wide-band FM wave.

Section 4.8: Demodulation of FM Signals

With the frequency modulator being a device that produces an output signal whose instantaneous frequency varies linearly with the amplitude of the input message signal

- It follows that ***For frequency demodulation*** we need a device whose output amplitude is sensitive to variations in the instantaneous frequency of the input FM wave in a linear manner too

Section 4.8: Demodulation of FM Signals

In what follows, we describe two devices for **frequency demodulation:**

1. One device, called a ***frequency discriminator***, relies on slope detection followed by envelope detection
2. The other device, called ***a phase-locked loop***, performs frequency demodulation in a somewhat indirect manner

Section 4.8: Demodulation of FM Signals - Frequency Discriminator

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR

- Frequency demodulators produce output voltage whose instantaneous amplitude is directly proportional to the instantaneous frequency of the input FM wave

$$g_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right).$$

- To extract the message signal contained in an FM signal, we can transfer the information from the angle to the magnitude by simply differentiating the FM signal

Section 4.8: Demodulation of FM Signals

- Since the derivate of a sinusoid results in multiplying the magnitude of the sinusoid by the derivate of its angle, the derivative of the above FM signal becomes

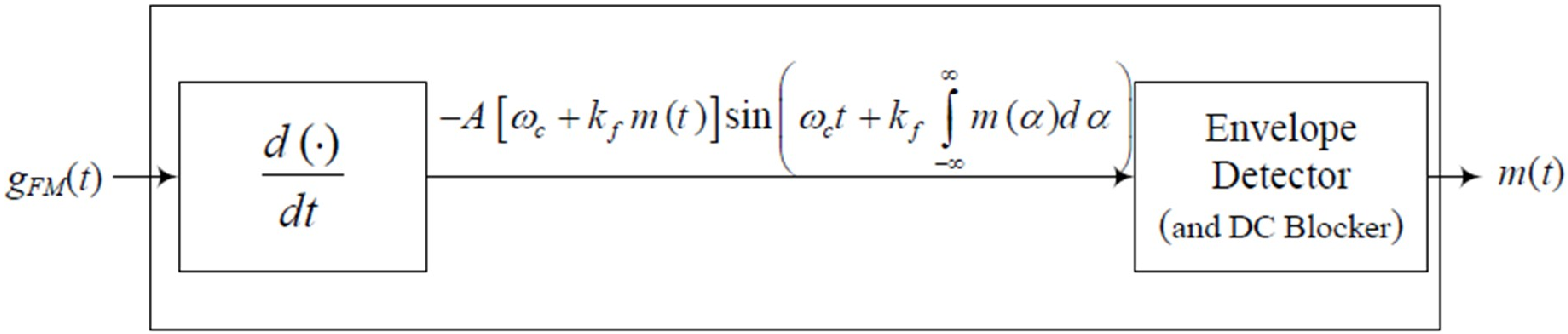
$$\frac{dg_{FM}(t)}{dt} = -A\left[\omega_c + k_f m(t)\right]\sin\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)$$

- So, the message signal of the above derivative is contained in the frequency of the sinusoid and also in its magnitude
- ***It is AM + FM signal***. Passing the derivative of the FM signal through an envelope detector will give the desired message signal at the output.

Section 4.8: Demodulation of FM Signals

- Therefore, the following block diagram is an FM demodulator

Signal Differentiation Frequency Demodulator



Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

Recall that the FM signal is given by

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

- ➢ The question to be addressed is: how do we recover the message signal m(t) from the modulated signal s(t)?
- We can motivate the formulation of a receiver for doing this recovery by noting that if we take the derivative of the above Eq

$$\frac{ds(t)}{dt} = -2\pi A_c[f_c + k_f m(t)] \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) \qquad (4.45)$$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

$$\frac{ds(t)}{dt} = -2\pi A_c[f_c + k_f m(t)] \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) \qquad (4.45)$$

We observe that the derivative is a band-pass signal with amplitude modulation defined by the multiplying term $[f_c + k_f m(t)]$

- Consequently, if $\mathbf{f_c}$ is large enough such that the carrier is not over modulated, then we can recover the message signal with an ***envelope detector*** in a manner similar to that described for AM signals

Section 4.8: Demodulation of FM Signals

1. **FREQUENCY DISCRIMINATOR – More details**

This idea provides the motivation for the ***frequency discriminator***, which is basically a demodulator that consists of ***a differentiator followed by an envelope detector***

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

- Differentiation corresponds to a linear transfer function in the frequency domain; that is $$\frac{d}{dt} \rightleftharpoons j2\pi f \qquad (4.46)$$
- In practical terms, it is difficult to construct a circuit that has a transfer function equivalent to the right-hand side of Eq. (4.46) for all frequencies
- Instead, we construct a circuit that approximates this transfer function over the band-pass signal bandwidth—in particular, for $f_c - (B_T/2) <= |f| <= f_c + (B_T/2)$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

- A typical transfer characteristic that satisfies this requirement is described by

$$H_1(f) = \begin{cases} j2\pi[f - (f_c - B_T/2)], & f_c - (B_T/2) \le |f| \le f_c + (B_T/2) \\ 0, & \text{otherwise} \end{cases} \quad (4.47)$$

- The transfer characteristic of this so-called ***slope circuit***

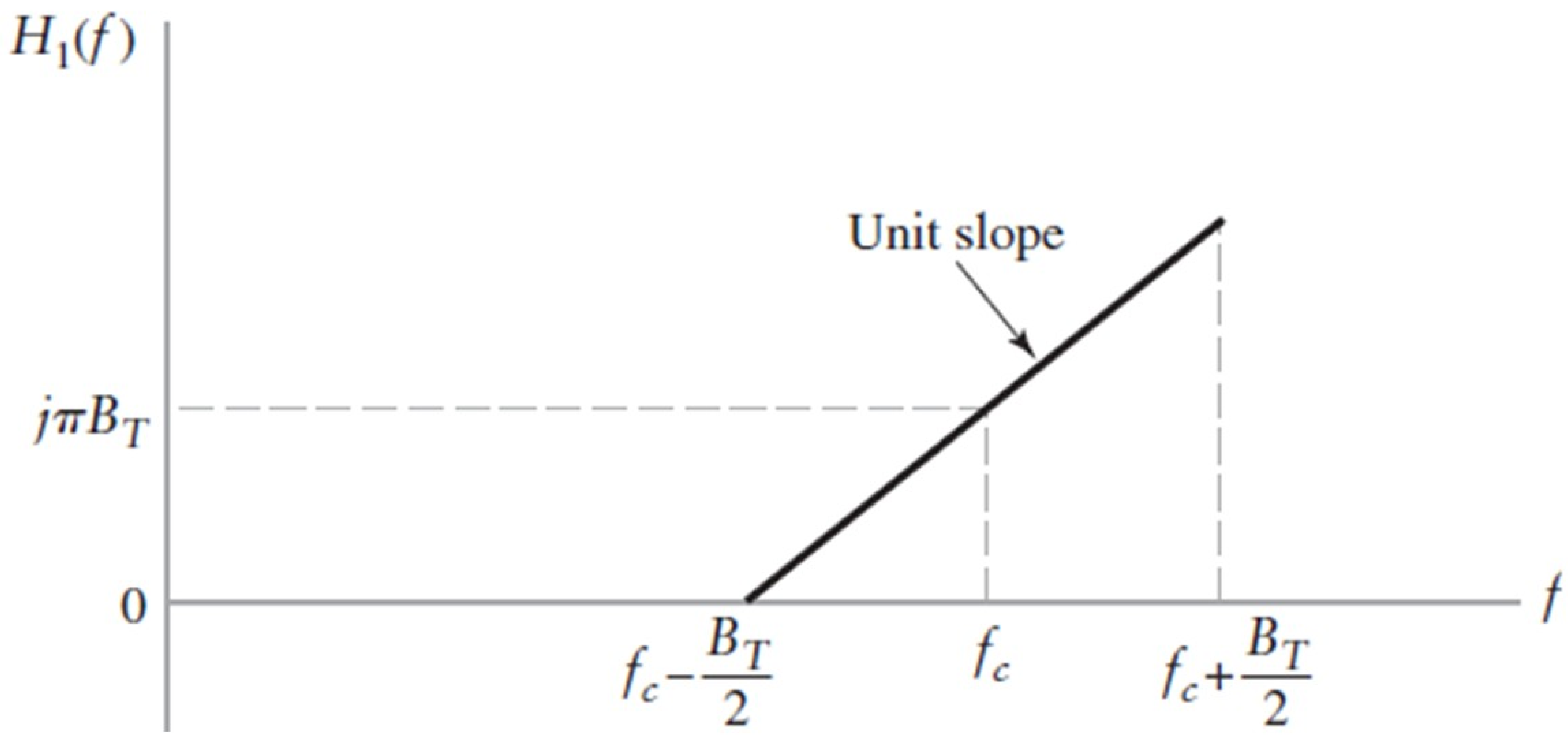


FIGURE 4.12 Frequency response of an ideal slope circuit.

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

- We find that the complex envelope of the FM signal

$$\tilde{s}(t) = A_c \exp\left(j2\pi k_f \int_0^t m(\tau)\, d\tau \right)$$

- The complex baseband filter (i.e., slope circuit) corresponding to Eq. (4.48) as

$$\tilde{H}_1(f) = \begin{cases} j2\pi[f + (B_T/2)], & -B_T/2 \le f \le B_T/2 \\ 0, & \text{otherwise} \end{cases} \qquad (4.49)$$

- Let $\tilde{s}_1(t)$ denote the complex envelope of the response of the slope circuit due to $\tilde{s}(t)$

$$\tilde{S}_1(f) = \frac{1}{2}\tilde{H}_1(f)\tilde{S}(f)$$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

- Then, according to the band-pass to low-pass transformation described in Chapter 3, we may express the Fourier transform of $\tilde{s}_1(t)$ as:

$$\tilde{S}_1(f) = \frac{1}{2}\tilde{H}_1(f)\tilde{S}(f)$$
$$= \begin{cases} j\pi\left(f + \frac{1}{2}B_T\right)\tilde{S}(f), & -\frac{1}{2}B_T \le f \le \frac{1}{2}B_T \\ 0, & \text{elsewhere} \end{cases} \qquad (4.50)$$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

1. Multiplication of the Fourier transform $\tilde{S}(f)$ by $j2\pi f$ is equivalent to differentiating the inverse Fourier transform $\tilde{s}(t)$ in accordance with Property 9 described in Eq. (2.33), as shown by

$$\frac{d}{dt}\tilde{s}(t) \rightleftharpoons j2\pi f\tilde{S}(f)$$

2. Application of the linearity property (i.e., Eq. (2.14)) to the nonzero part of $\tilde{S}_1(f)$ yields

$$\tilde{s}_1(t) = \frac{1}{2}\frac{d}{dt}\tilde{s}(t) + \frac{1}{2}j\pi B_T\tilde{s}(t) \qquad (4.51)$$

- Substituting Eq. (4.48) into (4.51), we get

$$\tilde{s}_1(t) = \frac{1}{2}j\pi A_c B_T\left[1 + \left(\frac{2k_f}{B_T}\right)m(t)\right]\exp\left(j2\pi k_f\int_0^t m(\tau)\,d\tau\right) \qquad (4.52)$$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR – More details

- Finally, the actual response of the slope circuit due to the FM wave *s(t)* is given by

$$s_1(t) = \mathrm{Re}[\tilde{s}_1(t)\exp(j2\pi f_c t)]$$

$$= \frac{1}{2}\pi A_c B_T\left[1 + \left(\frac{2k_f}{B_T}\right)m(t)\right]\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau + \frac{\pi}{2}\right) \quad (4.53)$$

- s1(t) is a ***hybrid*** modulated wave, exhibiting both ***amplitude modulation*** and ***frequency modulation*** of the message signal m(t). Provided that we maintain the extent of amplitude modulation

$$\left(\frac{2k_f}{B_T}\right)|m(t)|_{\max} < 1, \qquad \text{for all } t$$

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR

- The output of the envelope detector is given by

$$v_1(t) = \frac{1}{2}\pi A_c B_T\left[1 + \left(\frac{2k_f}{B_T}\right)m(t)\right] \qquad (4.54)$$

- The bias in v1(t) is defined by the constant term in Eq. (4.54)- namely, $\pi A_c B_T/2$
- ***To remove the bias, we may use a second slope circuit followed by an envelope detector of its own***
- This time, however, we design the slope circuit so as to have a *negative slope*

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR

- The output of this second configuration is given by

$$v_2(t) = \frac{1}{2}\pi A_c B_T\left[1 - \left(\frac{2k_f}{B_T}\right)m(t)\right] \quad (4.55)$$

- Accordingly,

$$\begin{aligned} v(t) &= v_1(t) - v_2(t) \\ &= cm(t) \end{aligned} \quad (4.56)$$

where c is a constant

Section 4.8: Demodulation of FM Signals

1. FREQUENCY DISCRIMINATOR

- The upper path of the figure pertains to Eq. (4.54).
- The lower path pertains to Eq. (4.55)
- The summing junction accounts for Eq. (4.56).

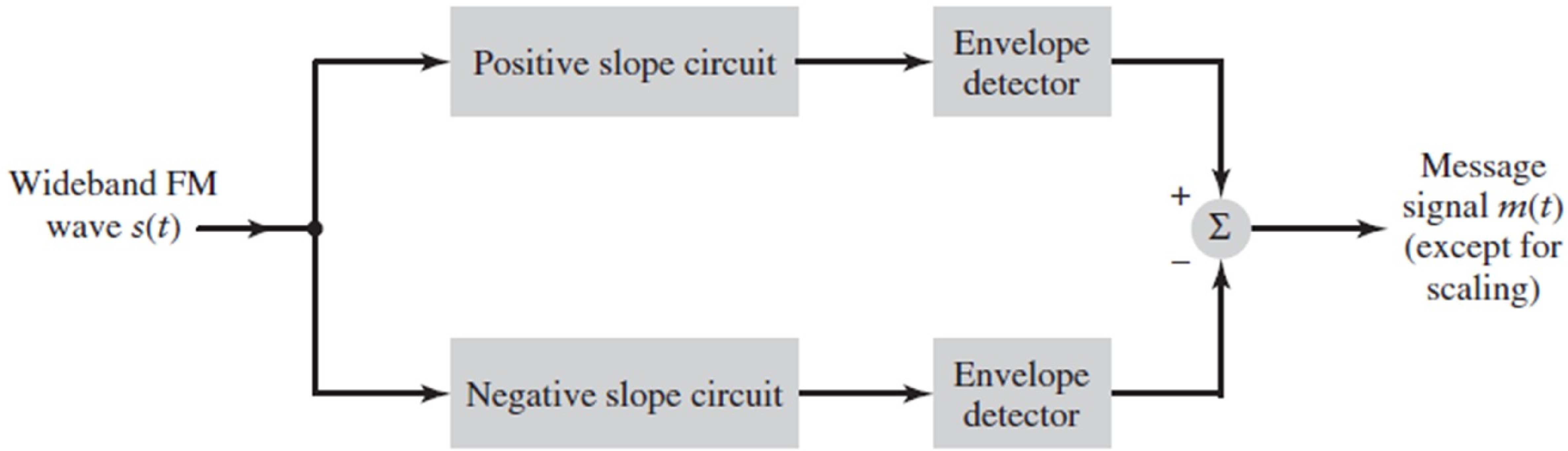


FIGURE 4.13 Block diagram of balanced frequency discriminator.

Section 4.8: Demodulation of FM Signals - **Phase-Locked Loop**

Section 4.8: Demodulation of FM Signals

The ***phase-locked loop*** is a feedback system whose operation is closely linked to frequency modulation

- It is commonly used for carrier synchronization, and indirect frequency demodulation
- It can be used also for frequency division/multiplication and frequency modulation

Section 4.8: Demodulation of FM Signals

Basically, the phase-locked loop consists of three major components:

I. *Voltage-controlled oscillator (VCO),* which performs frequency modulation on its own control signal

II. ***Multiplier***, which multiplies an incoming FM wave by the output of the voltage-controlled oscillator.

III. *Loop filter* of a low-pass kind, the function of which is to ***remove*** the high-frequency components contained in the multiplier's output signal and thereby shape the overall frequency response of the system

Section 4.8: Demodulation of FM Signals

A closed-loop feedback system

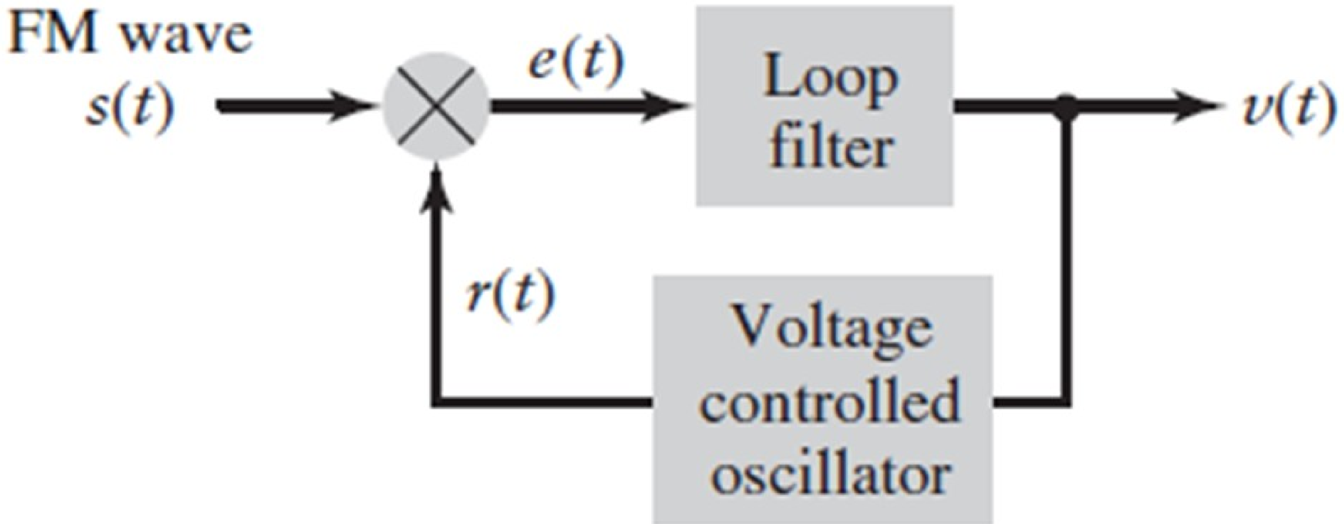


FIGURE 4.14 Block diagram of the phase-locked loop.

- The VCO is a ***sinusoidal generator*** whose frequency is determined by a voltage applied to it from an external source.

Section 4.8: Demodulation of FM Signals

To demonstrate the operation of the phase-locked loop as a frequency demodulator, we assume that the ***VCO has been adjusted so that when the control signal (i.e., input) is zero, two conditions are satisfied:***

1. The frequency of the VCO is set precisely at the unmodulated carrier frequency of the incoming FM wave s(t)
2. The VCO output has a 90-degree phase-shift with respect to the unmodulated carrier wave

Section 4.8: Demodulation of FM Signals

If the incoming FM wave is defined by

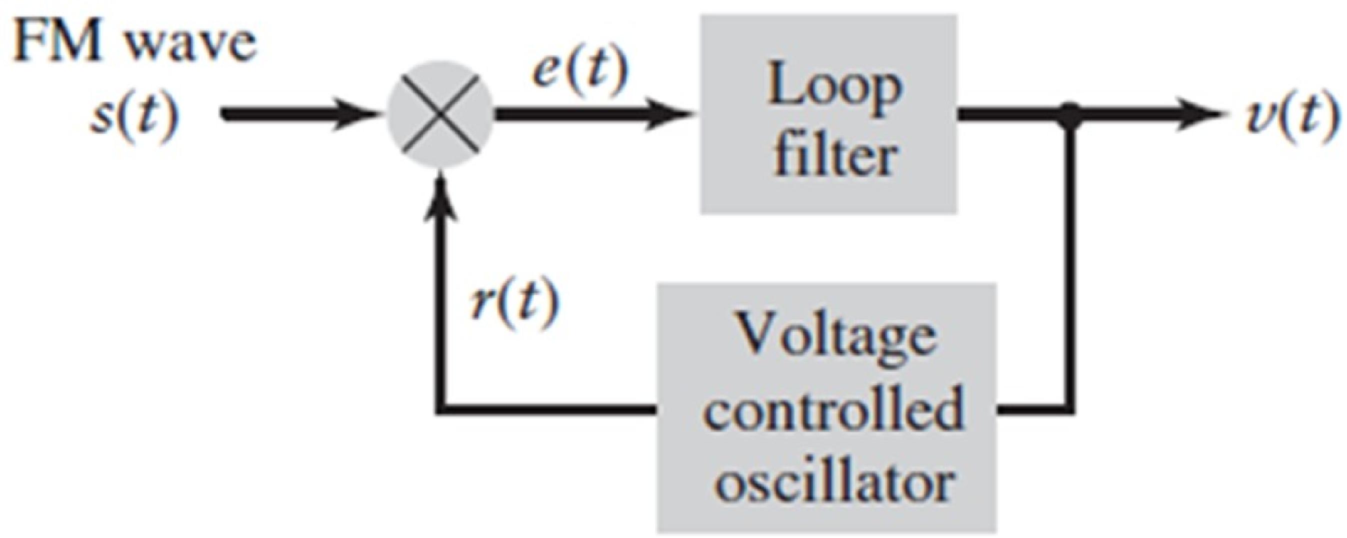
$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \tag{4.57}$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau \tag{4.58}$$

- We define the FM wave produced by the VCO as

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \tag{4.59}$$

$$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau \tag{4.60}$$



Section 4.8: Demodulation of FM Signals

The function of the feedback loop acting around the VCO is to adjust the angle $\phi_2(t)$ so that it equals $\phi_1(t)$, thereby setting the stage for frequency demodulation

◊ To develop an understanding of the phase-locked loop, it is desirable to have a model of the loop

◊ In what follows, we first develop a ***nonlinear*** model, which is subsequently ***linearized*** to simplify the analysis

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – non linear model

- According to Figure 4.16, the incoming FM signal $s(t)$ and the VCO output $r(t)$ are applied to the multiplier, producing two components:

1. A high- frequency component, represented by the *double- frequency* term

$$k_m A_c A_v \sin\left[4\pi f_c t + \phi_1(t) + \phi_2(t)\right]$$

2. A low- frequency component, represented by the *difference-frequency* term

$$k_m A_c A_v \sin\left[\phi_1(t) - \phi_2(t)\right]$$

where k_m is the *multiplier gain*, measured in volt^{-1}.

Section 4.8: Demodulation of FM Signals

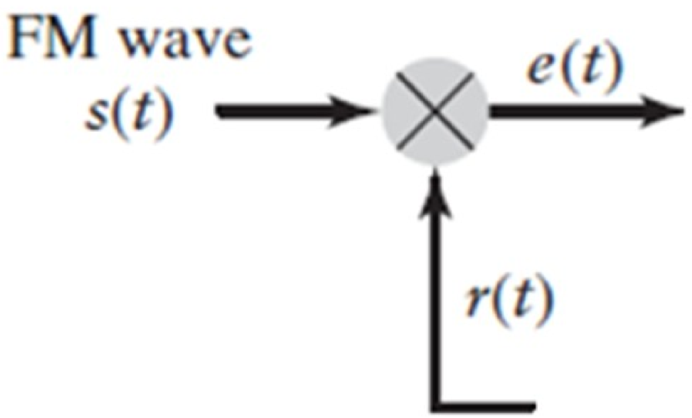
Phase-Locked Loop – non linear model

- *The loop filter in the phase-locked loop is a low-pass filter*, and its response to the high- frequency component will be negligible.

- Therefore, discarding the high-frequency component (i.e., the double- frequency term), the input to the loop filter is reduced to

$$e(t) = k_m A_c A_\upsilon \sin\left[\phi_e(t)\right] \qquad (4.63)$$

where $\phi_e(t)$ is the *phase error* defined by



$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$= \phi_1(t) - 2\pi k_\upsilon \int_0^t \upsilon(\tau)d\tau \qquad (4.64)$$

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – non linear model

- The loop filter operates on the input $e(t)$ to produce an output $v(t)$ defined by the convolution integral

$$\upsilon(t) = \int_{-\infty}^{\infty} e(\tau) h(t-\tau)\, d\tau \qquad (4.65)$$

where $h(t)$ is the impulse response of the loop filter.

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – non linear model

- Using Eqs. (4.64) to (4.65) to relate $\phi_e(t)$ and $\phi_1(t)$, we obtain the following nonlinear integro-differential equation as descriptor of the dynamic behavior of the phase-locked loop:

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(\tau)] h(t-\tau) d\tau \qquad (4.66)$$

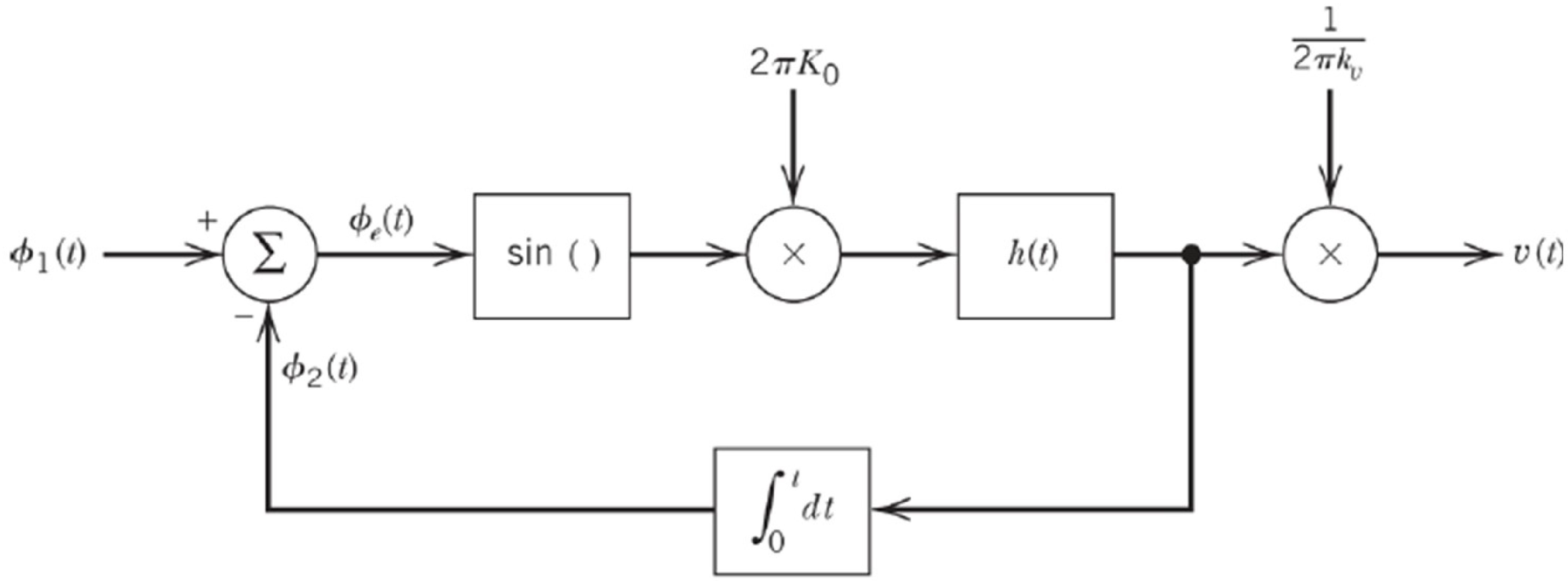
where K_0 is a loop-gain parameter defined by

$$K_0 = k_m k_\upsilon A_c A_\upsilon \qquad (4.67)$$

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – non linear model

- Equation (4.66) suggest the model shown in Figure 4.17 for a phase-locked loop.



- In this model we have also included the relationship between $v(t)$ and $e(t)$ as represented by Eqs. (4.63) and (4.65).

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Derivation of Eq. 4.66

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$= \phi_1(t) - 2\pi k_\upsilon \int_0^t \upsilon(\tau)d\tau \quad \left(\upsilon(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)d\tau,\ e(t) = k_m A_c A_\upsilon \sin[\phi_e(t)]\right)$$

$$= \phi_1(t) - 2\pi k_\upsilon \int_0^t \int_{-\infty}^{\infty} k_m A_c A_\upsilon \sin[\phi_e(k)]h(\tau-k)dkd\tau$$

$$= \phi_1(t) - 2\pi K_0 \int_0^t \int_{-\infty}^{\infty} \sin[\phi_e(k)]h(\tau-k)dkd\tau \quad (K_0 = k_\upsilon k_m A_c A_\upsilon)$$

$$= \phi_1(t) - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(k)] \int_0^t h(\tau-k)d\tau dk$$

$$\frac{\partial \phi_e(t)}{\partial t} = \frac{\partial \phi_1(t)}{\partial t} - \frac{\partial \phi_2(t)}{\partial t}$$

$$= \frac{\partial \phi_1(t)}{\partial t} - \frac{\partial 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(k)] \int_0^t h(\tau-k)d\tau dk}{\partial t}$$

(by using the Leibniz integral rule)

$$\frac{\partial}{\partial \alpha} \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)dx = \frac{\partial b(\alpha)}{\partial \alpha} f(b(\alpha),\alpha) - \frac{\partial a(\alpha)}{\partial \alpha} f(a(\alpha),\alpha) + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(x,\alpha)}{\partial \alpha} dx)$$

$$= \frac{\partial \phi_1(t)}{\partial t} - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(k)] \frac{\partial \int_0^t h(\tau-k)d\tau}{\partial t} dk$$

$$= \frac{\partial \phi_1(t)}{\partial t} - 2\pi K_0 \int_{-\infty}^{\infty} \sin[\phi_e(k)] h(t-k)dk$$

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – linear model

- When the phase error $\phi_e(t)$ is zero, the phase-locked loop is said to be in phase-lock. When $\phi_e(t)$ is at all times small compared with one radian, we may use the approximation

$$\sin\left[\phi_e(t)\right] \simeq \phi_e(t) \qquad (4.68)$$

which is accurate to within 4 percent for $\phi_e(t)$ less than 0.5 radians.

- We may represent the phase-locked loop by the linearized model shown in Figure 4.18*a*.

Section 4.8: Demodulation of FM Signals

Phase-Locked Loop – linear model

- We may represent the phase-locked loop by the linearized model shown in Figure 4.18*a*.

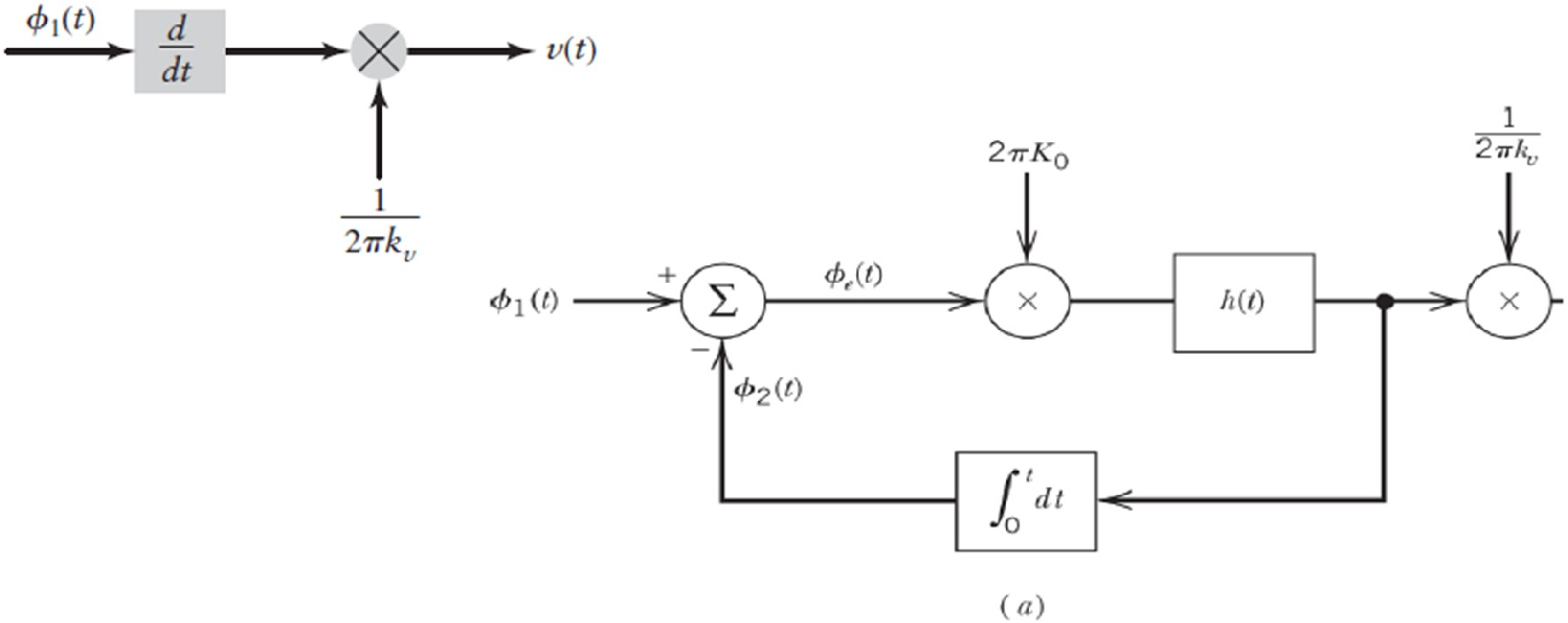

Figure 4.18 Models of the phase-locked loop. (*a*)Linearized model.